



บทความวิจัย

วงจรรขยายช่วงปฏิบัติงานเชิงเส้นสำหรับหม้อแปลงไฟฟ้ากระแสสลับแบบแปรผันเชิงเส้น (LVDT) ด้วยเทคนิคการชดเชยด้วยฟังก์ชันไฮเพอร์โบลิกแทนเจนต์ผกผัน

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บทคัดย่อ

บทความนี้ขอเสนอวงจรรขยายช่วงปฏิบัติงานเชิงเส้นสำหรับทรานสดิวเซอร์หม้อแปลงไฟฟ้ากระแสสลับแบบแปรผันเชิงเส้น (Linear Variable Differential Transformer; LVDT) ผลตอบสนองไม่เชิงเส้นจะถูกชดเชยด้วยสัญญาณที่ได้จากวงจรรฟังก์ชันไฮเพอร์โบลิกแทนเจนต์ผกผันด้วยการตั้งค่าอัตราขยายที่เหมาะสม โดยการกำหนดค่าอัตราขยายในส่วนต่าง ๆ ของวงจรรเพื่อชดเชยส่วนไม่เป็นเชิงเส้นลำดับที่ 3 และ 5 ส่งผลให้ช่วงปฏิบัติงานเชิงเส้นขยายกว้างขึ้นถึง 60 มิลลิเมตร และมีค่าความผิดพลาดสัมพัทธ์เชิงเส้น (Linearity Relative Error) ต่ำ (<2%) ตลอดช่วงการทำงานทั้งหมด ผลการจำลองจาก Pspice[®] แสดงให้เห็นถึงประสิทธิภาพของวงจรรที่เสนอสำหรับการขยายการกระจัดการทำงานเชิงเส้นของทรานสดิวเซอร์ LVDT โดยมีค่าความผิดพลาดสัมพัทธ์เชิงเส้นต่ำเมื่อเทียบกับเทคนิคอื่น

คำสำคัญ: วงจรรขยายช่วงปฏิบัติงานเชิงเส้น ทรานสดิวเซอร์แบบเหนี่ยวนำ หม้อแปลงไฟฟ้ากระแสสลับแบบแปรผันเชิงเส้น

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Linear Range Enhancement Circuit for Linear Variable Differential Transformer (LVDT) by Inverse Hyperbolic Tangent Function Compensation Technique

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Abstract

This paper proposes a Linear-Range Enhancement Circuit for Linear Variable Differential Transformer (LVDT). The nonlinearity is compensated with a signal obtained from an inverse hyperbolic tangent function circuit with appropriate gain settings. By configuring the gain in different sections of the circuit to eliminate the 3rd and 5th order term of nonlinearities, it results in enhancement of linearity range with low relative error over the entire operating ranges up to 60 mm with a low linearity relative error (<2%). The simulation results from Pspice[®] shows the efficiency of the proposed circuit for enhancing the linear operating displacement of the LVDT transducer with a low linearity error when compared to other techniques.

Keywords: Linear Range Enhancement Circuit, Inductive Transducer, LVDT

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1. Introduction

There are several types of high-resolution distance or position sensors used in engineering and industry. The most popular types of electrical sensors or transducers today are capacitive sensors, piezo-resistor sensors, opto-mechanical sensors, Micro-Electromechanical Systems (MEMS), including inductive transducers such as Linear Variable Differential Transformer (LVDT) [1] which is widely used especially for distance or position measurement applications that require high accuracy and stability. The internal structure of a LVDT transducer consists of primary coil positioned between the two secondary coils and a movable ferromagnetic core contained inside a cylindrical container rod. As shown in Figure 1 (a), when an exciting AC signal is applied to the primary core, it produces an electrical signal induced at the secondary coils (V_d) which has the same frequency as the excitation signal but whose magnitude changes depend on the position of the moving magnetic core. This behavior is similar to a modulator circuit that controls the modulation signal magnitude with the position of the magnetic core. Thus, when demodulating the inductive signal, the displacement of the moving magnetic core (l) is obtained. The commercial LVDT measuring range is about ± 100 microns to ± 25 cm. The excite AC voltage ranges from 1 to 24 Vrms, with frequencies range from 50 Hz to 20 kHz.

Although the LVDT transducer has advantages of providing high resolution measurement results and a simple operation compared to other transducers, the linear range of response signal to the core displacement is limited to about 25–30% of the total operating displacement, as shown in Figure 1 (b).

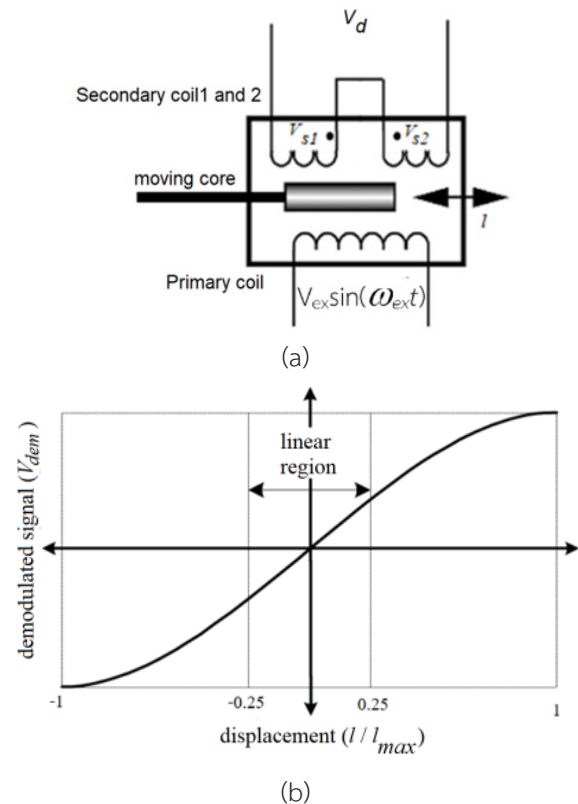


Figure 1 (a) A structural model of LVDT transducer, (b) Demodulated signal response to displacement of moving core.

This limits its application; otherwise it needs a complex system to determine the displacement from the non-linear response signal to expand the measurement operating range, such as the use of digital processing techniques to determine the frequency domain response from FFT (fast Fourier Transform) [2], the application of an analog adaptive circuit controlled by fuzzy Proportional Integral Derivative (PID) controller [3], a digitally-controlled excitation-feedback input phase analysis technique [4], including the use of pseudo-network in the fuzzy Proportional Integral Derivative. The signal processing technique to compensate for the

nonlinearity of the moving core displacement response signal [5] makes the system complex, and expensive. Another technique uses the primary winding excitation signal to provide a linear response when moving the magnetic core. But the measurement complexity and accuracy depend on the generation of excitation signals, such as triangular inputs as excitation signals [6], making them unsuitable for high resolution applications.

The linearity range enhancement techniques are using analog mathematic function circuit, which has the advantage of small and uncomplicated circuit. Although there are limitations in accuracy as the magnetic axis travel approaches the maximum travel distance (l_{max}). For example, using the nonlinearity compensation technique by estimating it with the binomial inverse function [8], this uses a multiplication circuit as a part of the circuit. The accuracy of the measurement results will depend on the efficiency of the multiplier circuit. High-efficiency multiplier circuits are expensive and require appropriate configurations. For the compensation of nonlinearities by the logarithm approximation technique [7], the logarithmic difference ($1/2(\ln|1+x| - \ln|1-x|)$) results a simple circuit and does not require a multiplier circuit. But the nonlinearity compensation is limited by the nonlinearity of the signal synthesized from the two logarithmic circuits. This makes it difficult to adjust the circuit settings for good results.

This paper presents a linear enhancement circuit for LVDT by generating a nonlinear signal with a scalable inverse hyperbolic tangent function, compensating back to LVDT demodulated (V_{dem}) signal. This makes the configuration convenient

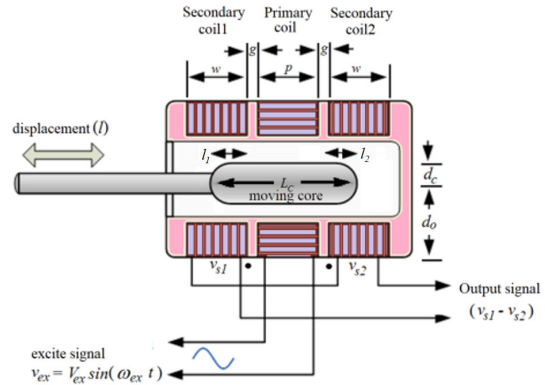


Figure 2 The internal structural model of LVDT.

and simple. Part 2 describes the basic structure and working principle of an LVDT transducer. The proposed techniques to improve linearity range and the proposed circuit are mentioned here. Parts 3 show simulation with Pspice[®] Electronic Circuit Simulator. The last part is a summary of observations and future developments.

2. Materials and Methods

2.1 Structure and working principle of LVDT

The LVDT transducer operates similarly to the operation of a transformer based on the electromagnetic induction principle, which consists of the primary coil, two identical secondary coils and the ferromagnetic moving core as shown in Figure 2.

The two secondary coils are connected in opposite directions to produce a difference signal ($v_{s1} - v_{s2}$) of the inductive field whose amplitude changes with the position of the moving core, when excitation by an alternating current (v_{ex}) signal. For example, a $V_{ex}\sin(\omega_{ex}t)$ sinusoid signal with an angular frequency (ω_{ex}) and magnitude (v_{ex}) is an excitation signal on the primary coil. This induces a voltage induced on the secondary cores with a

signal frequency of ω_{ex} and amplitude as shown in Equations (1) and (2) [8].

$$v_{s1} = \frac{2\pi^2 \omega_{ex} V_{ex} n_p n_s (2l_2 + p)}{10^7 w L_C Z_p \ln(d_o / d_c)} l_1^2 \quad (1)$$

$$v_{s2} = \frac{2\pi^2 \omega_{ex} V_{ex} n_p n_s (2l_1 + p)}{10^7 w L_C Z_p \ln(d_o / d_c)} l_2^2 \quad (2)$$

where

p, w are length of primary and secondary coils.

d_o, d_c are radius of the coil and the moving core.

n_p, n_s are number of turns of primary and secondary coils.

l_c is length of moving core.

Z_p is impedance of primary coil.

l_1, l_2 are the lengths of the moving core sliding into the secondary coil S_1 and S_2 , respectively.

Therefore, the different signal ($v_{s1} - v_{s2}$) is equal to Equation (3).

$$v_{s1} - v_{s2} = K_1 l (1 - K_2 l^2) \quad (3)$$

From the Equation (4)–(7).

$$K_1 = \frac{8\pi^2 \omega_{ex} V_{ex} n_p n_s (p + 2g + l_0)}{10^7 w L_C Z_p \ln(d_o / d_c)} l_0 \quad (4)$$

$$K_2 = \frac{1}{(p + 2g + l_0) l_0} \quad (5)$$

$$l_0 = \frac{l_1 + l_2}{2} \quad (6)$$

$$l = \frac{l_1 - l_2}{2} \quad (7)$$

From the structure of the transducer, LVDT can measure the distance traveled (Displacement; l) of

the moving core. The difference signal between the induced electrical signals at the two secondary coils ($v_{s1} - v_{s2}$) then feed through a demodulator circuit to obtain the demodulated signal (v_d) whose amplitude of the signal is vary depending on displacement of moving core. In practice, the length p is greater than the gap g ($p \gg g$). From Figure 2, the length of the axis of travel (l_c) can be estimated as $l_c = 3p$. Therefore, the amplitude of the demodulated signal (v_d) is therefore equal to Equation (8).

$$v_d = k_s l (1 - K_n l^2) = k_s l - k_s K_n l^3 \quad (8)$$

From the Equation (9) and (10).

$$k_s = \frac{8\pi^2 \omega_{ex} V_{ex} n_p n_s}{10^7 Z_p \ln(d_o / d_c)} \frac{2p}{3w} \quad (9)$$

$$K_n = \frac{1}{2p^2} \quad (10)$$

and, can be modeled as Equation (11).

$$K_n = \frac{k_n}{(l_{max})^2} \quad (11)$$

The k_s is the sensitivity constant (Volts/meter), K_n is the nonlinearity coefficient ($1/\text{meter}^2$) and k_n is the nonlinearity coefficient (unitless) which is relative to the maximum displacement length (l_{max}). For commercial transducers LVDTs, the moving core displacement length is typically $-20 \text{ mm} < l < 20 \text{ mm}$ ($|l_{max}| = 20 \text{ mm}$), a nonlinearity coefficient in the range of $0.1 < k_n < 0.3$ or equal to $250 < K_n < 750$ ($1/\text{meter}^2$) for $|l_{max}| = 20 \text{ mm}$.

The non-linearity effect limits the accuracy of measured displacement length of transducers. The displacement voltage relative error (ε_{r_vd}) can

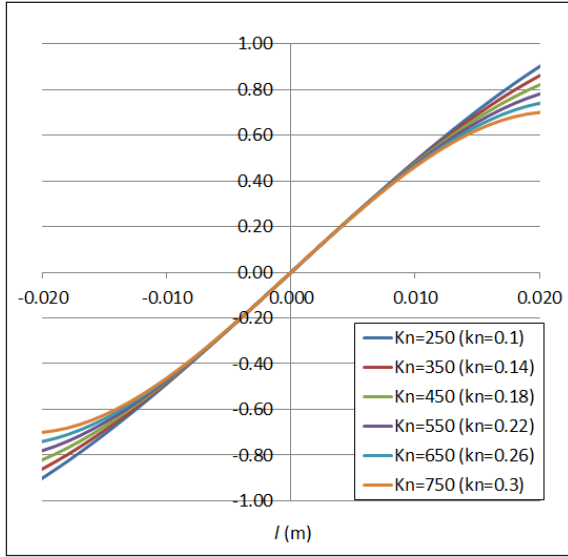


Figure 3 The differential signal (v_d) of two secondary cores of LVDT.

be defined as shown in equation (12).

$$\varepsilon_{r_vd}(\%) = \frac{|v_d - v_{d_linear}|}{v_{d_linear}} \times 100 \quad (12)$$

From equation (8), the v_{d_linear} is $k_s l$, therefore, ε_{r_vd} equals to Equation (13).

$$\varepsilon_{r_vd}(\%) = k_s K_n l^2 \quad (13)$$

The calculation result of the demodulated signal (v_d) of LVDT per Equation (8) when given k_s equals to 50 (V/meter) is shown in Figure 3.

2.2 Proposed Linearity Range Enhancement Technique

From Figure 3, it can be seen that the linear operating range of the transducer is limited. Especially for transducers with high nonlinearity coefficient (K_n), the result will be worse when the

moving core is close to the maximum displacement limit. To increase the operating displacement accuracy, this paper presents a linearity range enhancement technique by generating the nonlinear part from inverse hyperbolic tangent which can be approximated by Taylor's series expansion as shown in Equation (14).

$$\operatorname{arctanh}(x) \approx x + \frac{x^3}{3} + \frac{x^5}{5} + \dots ; |x| < 1 \quad (14)$$

With a proper setting, the generated signal can be used to compensate the nonlinear portion of the LVDT (last term of Equation (8)). The proposed technique is shown in Equation (15).

$$v_{LE} = av_d + b(\operatorname{arctanh}(cv_d)) \quad (15)$$

where, a and b are signal gain of LVDT signal and inverse hyperbolic tangent compensation generated signal, respectively. The parameter c is used for maximizing the linear operation range. Substitute Equation (8), (14) and (15), the linearity range enhanced signal (v_{LE}) can be approximated as shown in Equation (16).

$$\begin{aligned} v_{LE}(l) \approx & l(a + bc)k_s \\ & + l^3 \left(b \left(\frac{c^3 k_s^3}{3} - ck_s K_n \right) - ak_s K_n \right) \\ & + l^5 \left(-bc^2 k_s^2 \right) \left(\frac{ck_s K_n}{3} - ck_s \left(\frac{c^2 k_s^2}{5} - \frac{2K_n}{3} \right) \right) \\ & + l^7 \left(bc^3 k_s^3 \right) \left(c^2 k_s^2 \left(\frac{c^2 k_s^2}{7} - K_n \right) + K_n^2 \right) + \dots \end{aligned} \quad (16)$$

and $|v_d(l_{max})| < 1$.

To eliminate the 5th order term, set c equal to Equation (17).

$$c = \frac{\sqrt{5K_n}}{k_s} \quad (17)$$

To keep the v_{LE_linear} close to v_{d_linear} therefore, $(a + bc) = 1$. Then, a and b can easily be found to remove out the 3rd order term by setting c as equal Equation (17). The result is shown in Equation (18).

$$a = \frac{2}{5} \text{ and } b = \frac{3}{5c} = \frac{3}{5} \frac{k_s}{\sqrt{5K_n}} \quad (18)$$

Substitute a , b and c back to Equations (15) and (16), the results of v_{LE} will be as shown in Equations (19) and (20), respectively.

$$v_{LE} = 0.4v_d + 0.268 \frac{k_s}{\sqrt{K_n}} \left(\operatorname{arctanh} \left(2.236 \frac{\sqrt{K_n}}{k_s} v_d \right) \right) \quad (19)$$

and,

$$v_{LE}(l) = lk_s - l^7 \left(\frac{9k_s K_n^3}{7} \right) + \dots, \quad (20)$$

As Equation (12), therefore, the linearity range enhancement displacement voltage relative error (ϵ_{r_VLE}) equals to Equation (21).

$$\epsilon_{r_VLE} (\%) = \frac{|v_{LE}(l) - lk_s|}{lk_s} \times 100 \approx l^6 \left(\frac{9K_n^3}{7} \right) \times 100 \quad (21)$$

Note that when comparing Equation (21) and Equation (13), because $l < 1$, the relative error of

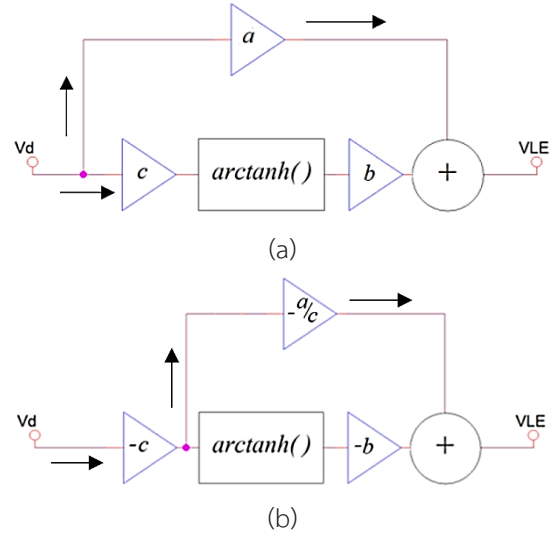


Figure 4 The proposed linearity range enhancement block diagram (a) as Equation (15) and (b) modified for circuit implementation.

proposed technique (ϵ_{r_VLE}) is much lower than the original one (ϵ_{r_vd}).

2.3 Circuit Implementation and Simulation

From Equation (15), the proposed circuit can be shown in the block diagram as shown in Figure 4. Its structure is simple and does not require any expensive multiplication circuits. The circuit consists of three amplifier circuits, an inverse hyperbolic tangent circuit and summing circuit. The input signal (v_d) of the circuit is the demodulated signal of the difference induced signal of secondary coils, when applying the excite signal to the primary coil and move the core of the LVDT transducer.

The inverse hyperbolic tangent ($\operatorname{arctanh}()$) circuit can be implemented in several ways. In this paper, the circuit will be built from an OpAmp (U3) and a bipolar based OTA (U1) [9] that acts as a feedback circuit as shown in Figure 5. Its transfer

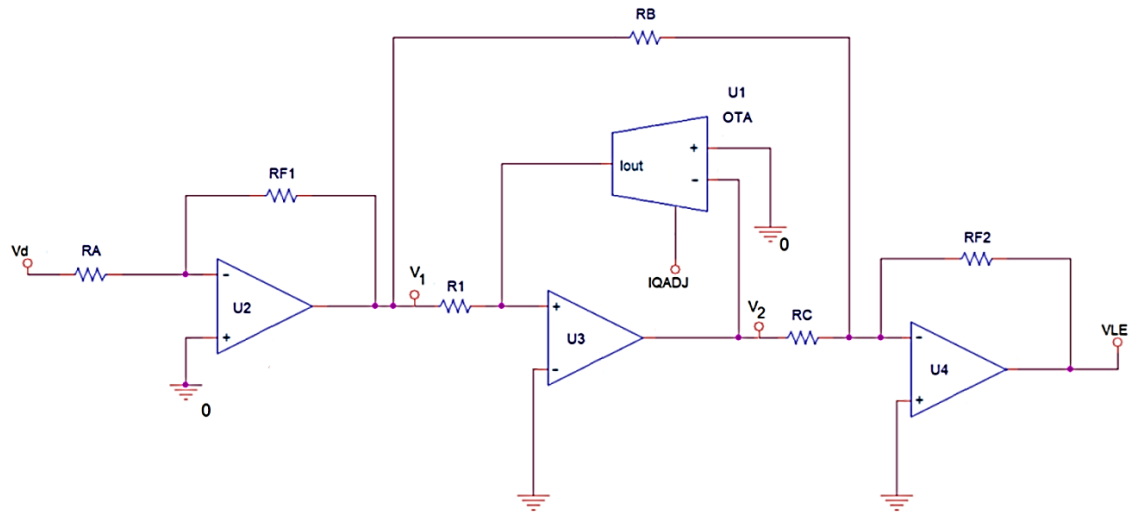


Figure 5 The proposed linearity range enhancement circuit.

function shows in Equation (22),

$$v_2 = 2V_T \operatorname{arctanh}\left(\frac{v_1}{R_1 I_{QADJ}}\right) \quad (22)$$

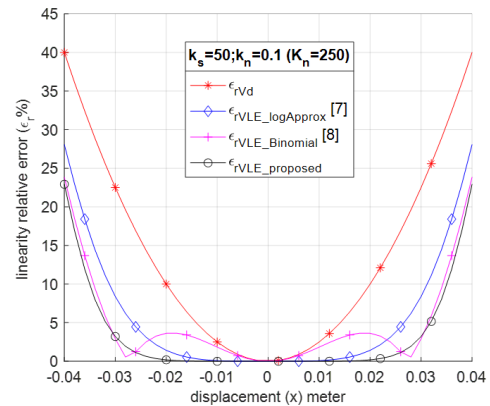
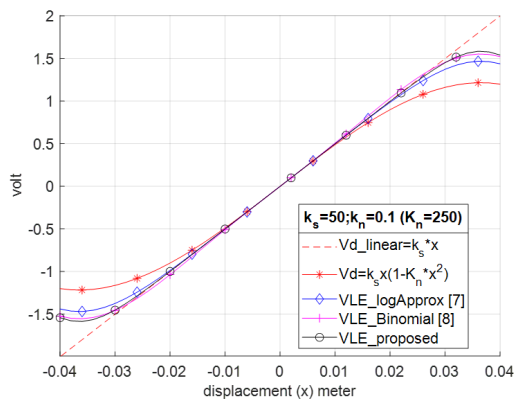
Where, V_T is thermal voltage and equal to kT/q with equal to 26 mV at room temperature (27 °C) which k is Boltzmann's constant, T is temperature (Kelvin) and q is electron charge. To simplify Equation (22), R_1 and I_{QADJ} are set as 50 Ohm and 20 mA, respectively. The OpAmp (U2) works as amplifier (c) in Figure 4(b), while amplifier $(-a/c)$ and $(-b)$ are parts of negative summing amplifier (U4). The gain of amplifier $(-a/c)$ and (b) can be set by R_B , R_C and R_{F2} , while gain of amplifier (c) can be set by R_A and R_{F1} .

3. Results

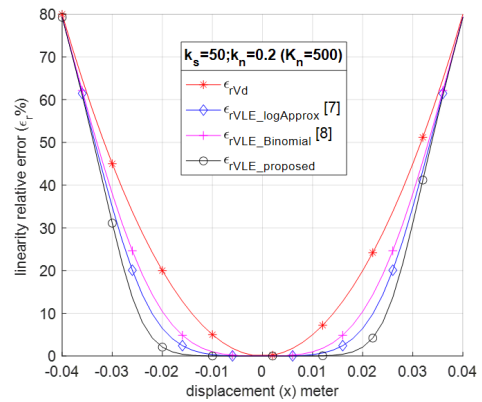
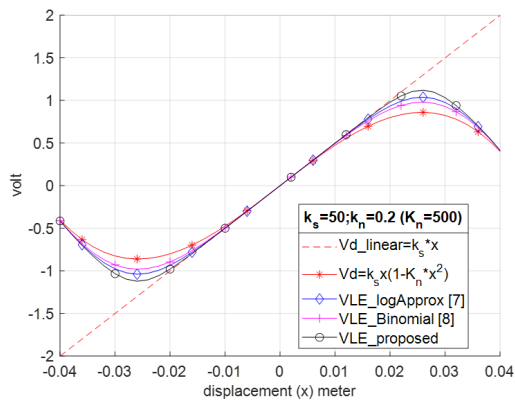
In order to demonstrate the effectiveness of the proposed technique, the Low-cost Linear Range Enhancement by logarithm approximation

technique [7] ($v_{LE_logApprox}$) and inverse transfer characteristic using binomial series approximation technique [8] are computed. The comparison of proposed technique as shown in Equation (18) with previous two techniques and non-compensated (v_d) are shown in Figure 6. It shows that the proposed technique provides the better linear range enhancement compared with [7], [8] and non-compensated v_d . The linear operation displacement was enlarged several times compared to the non-compensated one. In case of the relative error ($\epsilon_{r_VLE\%}$) is restricted to lower than 2%, the enhanced linearity displacement range are ± 31 , ± 22 and ± 18 mm for LVDT with $k_s = 50$ v/m and $k_n = 0.1$ ($K_n = 250$ 1/m²), 0.2 ($K_n = 500$ 1/m²) and 0.3 ($K_n = 750$ 1/m²), respectively.

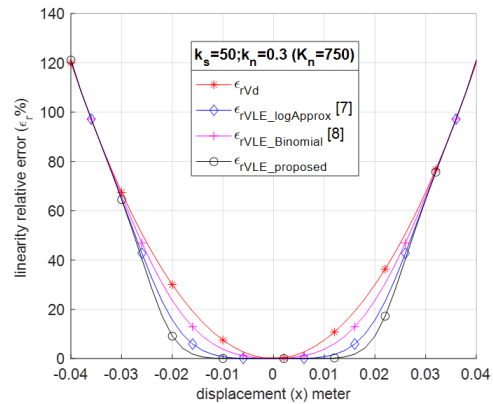
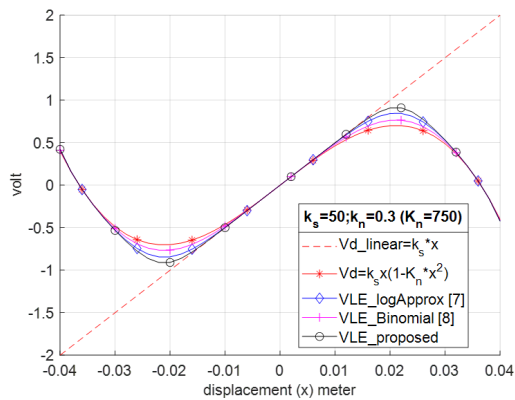
Figure 7 shows the Pspice[®] simulation result of proposed technique compared with [7] and non-compensated v_d for $k_n = 0.1$, 0.2 and 0.3. The relative linearity error ($\epsilon_{r_VLE\%}$) shows in Figure 8. In case of the relative error ($\epsilon_{r_VLE\%}$) is restricted



(a)



(b)



(c)

Figure 6 The comparison of computation linearity range enhanced result and displacement linearity relative error (ϵ_r ,%) of proposed technique with [7], [8] and non-compensated (v_d). (a) $k_n = 0.1$, (b) $k_n = 0.2$ and (c) $k_n = 0.3$.

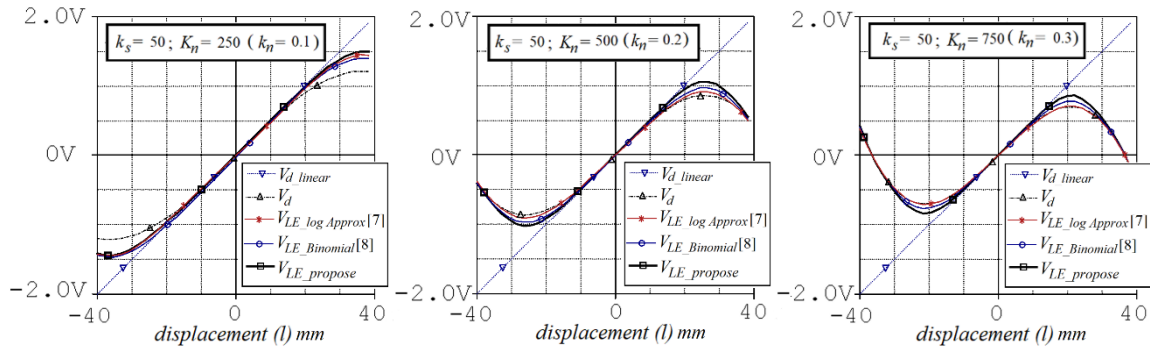


Figure 7 The Pspice[®] simulation result of proposed technique compares with [7], [8] and non-compensated (v_d) for LVDT ($k_s = 50$ v/m), (a) $k_n = 0.1$, (b) $k_n = 0.2$ and (c) $k_n = 0.3$ ($1/m^2$)

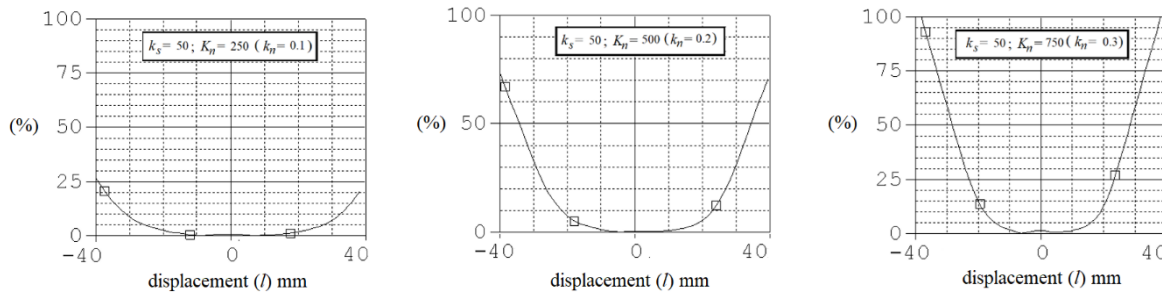


Figure 8 The Pspice[®] simulation result of linearity relative error (%) of proposed technique (a) $k_n = 0.1$, (b) $k_n = 0.2$ and (c) $k_n = 0.3$

to lower than 2%, the enhanced linearity displacement range are ± 20 , ± 15 and ± 12 mm, for $k_n = 0.1, 0.2$ and 0.3 , respectively.

4. Discussion and Conclusion

This paper proposes a linearity range enhancement for LVDT using inverse hyperbolic tangent approximation to compensate the non-linear part. Applying the function analytic, the linear range can be extended to be close to asymptotes. The proposed linear enhancing circuit requires only 3 OpAmps, an OTA and a small number of passive elements. The efficiency of the proposed circuit depends on the accuracy of the signal obtained

by the inverse hyperbolic tangent circuit. Under the condition of the relative error $< 1\%$, The calculation result for LVDT ± 35 mm stroke range with $k_s = 50$ v/m and $k_n = 0.1$ shows that the proposed method extends a linear range to ± 25 mm. When comparing it with the logarithm approximation technique [7], the binomial inverse function technique [8] and non-compensation LDVT, the linear range is widened 0.3, 2.5 and 3.1 times, respectively. This finding coincides with the PSpice simulation.

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