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บทความวิจัย

An Improved Whale Optimization Algorithm for Vehicle Routing Problem with Time Windows

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Abstract: The vehicle routing problem with time windows (VRPTW) is a pivotal problem in logistics operation management which attempts to establish routes for vehicles to deliver goods to customers. The objective of VRPTW is to find the optimal set of routes for a fleet of vehicles in order to serve a given set of customers within time window constraints. As the VRPTW is known to be NP-hard combinatorial problem, it is hard to be solved in reasonable computational time. Therefore, this paper proposes the modification of the whale optimization algorithm with local search to solve the VRPTW. The local search comprised 2-Operator and single insertion for solution improvement. Furthermore, the 2-Operator is used after the exploration phase and single insertion in the exploitation phase. The computational experiments were applied to Solomon's instance that included small to large size problems. The experiment results show that the average gap of the total distance between the Best Known Solution (BKS) and the proposed solutions is within 5.82%. In addition, the best solution was found 29 out of 56 instances that is better than the PSO at 1.09%. This shows that this proposed provides a minimum value and outperforms other metaheuristics approaches.

Keywords: Whale Optimization Algorithm; Vehicle Routing Problem; Time Constraints

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1. Introduction

Vehicle Routing Problem (VRP) is an important problem in the field of transportation, distribution, and logistics which was by Dantzig and Ramser [1] in their study on the truck dispatching problem. VRP attempts to find routes for a number of vehicles to minimize total transportation costs while satisfying the demands of customers. A well-known variant of VRP where there exists limits capacity for each vehicle is called “Capacitated Vehicle Routing Problem (CVRP)”. Primary constraints of CVRP include each route must start and return at the depot, each customer must be visited by only one vehicle and the total demand for the route of each vehicle must not exceed the vehicle capacity.

VRP has extensive variants, for example, Multi-depot vehicle routing problem (MDVRP) [2] , Periodic vehicle routing problem (PVRP) [3], Split delivery vehicle routing problem (SDVRP) [4] and Heterogeneous VRP [5]. One of the most common extensions of traditional VRP is time windows constraint (VRPTW) where vehicles must service all the customers during their pre-specified time intervals without violating capacity constraints. Time windows are divided into two types: soft and hard time windows. Soft time windows allow the vehicles to arrive prior to or later than their corresponding earliest and latest time, respectively. If the vehicle arrives before the starting time windows, it must wait until the time has been reached. On the other hand, if it arrives later than the finishing time windows, a penalty cost is incurred. Hard time windows constraints do not allow such relaxation.

VRPTW is NP-hard problem and has been intensively studied. Early solution techniques applied ware exact method; Branch-and-cut [6], Column generation [7] and Branch-and-price [8]. With the limitation of exact algorithms, many researchers developed various approximation algorithm to solve the problem which are capable of finding near optimal solution within reasonable computing time. The approximation method for solving VRPTW and its variants are classified into two categories: classical heuristic and meta-heuristic. The well-known meta-heuristic algorithms include Tabu search (TS) [9], Simulated annealing (SA) [10], Scatter search (SS) [11], Particle swarm optimization (PSO) [12], Artificial bee colony (ABC) [13]. Some works on VRPTW apply hybrid heuristics, Ant colony optimization with tabu search (ACO-TS) [14], Ant colony system (ACS) with Brain storm optimization (BSO) [15].

The whale optimization algorithm is newly proposed population-based metaheuristics described by Mirjalili and Lewis [16]. This algorithm mimics the unique foraging behavior of humpback whales to catch the prey. The WOA has been used to solve various filed in engineering problem. For example, Prakash and Lakshminarayana [17] used WOA for finds optimal sizing and placement of

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capacitors in radial distribution network. Yan et al. [18] solved multi-objective water resource allocation optimization model by WOA in real data. Abdel-Basset et al. [19] proposed a new the WOA with local search strategy for tacking the permutation flow shop scheduling problem. Khalilpourazari et al. [20] proposed an robust possibilistic programing for Multi-product economic order quantity model (EOQ) and solved by WOA with Water Cycle Algorithm. Dewi and Utama [21] proposed Hybrid WOA with TS and local search to minimize the distribution cost comprised of fuel cost, emission carbon cost and vehicle -used cost in Green VRP. Jafari-Asi et al. [22] Proposed WOA for design shape of the labyrinth spillway for dam. Zhang et al. [23] proposed variable neighborhood discreate whale optimization algorithm for traveling salesman problem. For combinatorial optimization problem, there are relatively few studies using the WOA. However, several previous studies have shown that WOA has been effectively used in solving problem.

The main contributions of this paper can be summarized as follows, First, this paper proposed the WOA with local search for solving the VRPTW. The objective function is a minimization of the total distance. Second, show the performance of the WOA that solve different size problem. We believe that our methodology and main contribution is to propose that is new and original.

The remainder of this paper is organized as follows. A mathematical programming model of the problem is presented in Section 2. Section 3 presents the whale optimization algorithm and local search. Section 4 shows and describes the computational experiment in this work and Section 5 provides conclusions and future works.

2. Mathematical Model

In this section, we present a mathematical programming model for our problem. We assume that vehicle are homogeneous fleet. Each customer has a time windows for receiving service. Let us define a Graph, $G=(V,A)$ as a transportation network where $V=\{0,1,2,...,N\}$ is a vertex set (vertex 0 corresponds to the depot), and the set of vertexes $i=\{1,2,...,N\}$ corresponds to the customers where N is the number of customers. In this graph, a distance d_{ij} and travel time t_{ij} are associated with every arc $(i,j)\in A$, where A is arcs set. Each customer i has a demand q_i, a service time s_i and a time windows (e_i, l_i), where e_i and l_i are the earliest time and the latest time to start the service of customer i. In the case where the vehicle to serve customer i arrives prior to e_i, it must wait at customer i location until time e_i, Note that $d_0=0$, $e_0=0$ and $l_0=T$, where T is the latest time in which vehicles can return to the depot.

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Set and Indices

i, j Index for customers.

k Index for vehicles.

N Set of customers.

K Set of vehicles.

Parameters

d_{ij} the distance from customer i to j.

t_{ij} the travel time from customer i to j.

q_i the demand of customer i.

Q_k the capacity of vehicle k.

s_i the service time of customer i.

e_i the earliest arrival time for customer i.

l_i the latest arrival time for customer i.

t_i the arrival time of vehicle at customer i.

w_i the waiting time of vehicle at customer i.

M the large positive number.

Decision variable

$$X_{ijk} = \begin{cases} 1 & \text{If the vehicle travel from } i \text{ to } j \\ 0 & \text{Otherwise} \end{cases}$$

A_i Arrival time of customer i

Objective function

$$Min \sum_{k\in V}\sum_{i\in N}\sum_{j\in N} d_{ij} X_{ijk} \quad (1)$$

Subject to

$$\sum_{k\in K}\sum_{i\in N} x_{ijk} = 1 \qquad \forall j \in N, i \neq j \quad (2)$$

$$\sum_{k\in K}\sum_{j\in N} x_{ijk} = 1 \qquad \forall i \in N, i \neq j \quad (3)$$

$$\sum_{i\in N}\sum_{j\in N} q_i X_{ijk} \leq Q_k \qquad \forall k \in K \quad (4)$$

$$\sum_{j\in N, j\neq 0} X_{0jk} \leq 1 \qquad \forall k \in K \quad (5)$$

$$\sum_{j\in N} X_{0jk} = \sum_{i\in N} X_{i0k} \qquad \forall k \in K \quad (6)$$

$$A_i + s_i + t_{ij} + w_i - M(1 - X_{ijk}) \leq A_j \qquad \forall i, j \in N, \forall k \in K \quad (7)$$

$$e_i \leq A_i + w_i \leq l_i \qquad \forall i \in N \quad (8)$$

$$w_i = max\{e_i - t_i, 0\} \qquad \forall i \in N \quad (9)$$

$$t_i \leq A_i \qquad \forall i \in N \quad (10)$$

$$A_i \geq 0 \qquad \forall i \in N \quad (11)$$

$$x_{ijk} \in \{0,1\} \qquad \forall i, j \in N, \forall k \in K \quad (12)$$

The objective function (1) minimizes the total distance. Constraint (2) and (3) guarantee that each customer is served by exactly one vehicle. Constraint (4) is the capacity constraint, which states that for each route, the accumulated demand from the customers who are served by a vehicle must not exceed the vehicle capacity. Constraint (5)-(6) guarantee that the vehicles start from and return to the depot. Constraint (7) specifies the time relationship between two customers who are served by the same vehicle. Constraint (8)-(10) are related to time windows. Constraint (11)-(12) are decision variables.

3. Whale Optimization Algorithm

The whale is predator that never sleep because it must breathe from the surface to ocean. The social behavior of whale is live alone or group. However, they are mostly observed in groups. Humpback is one the biggest baleen whale. The interesting point about the humpback whales is special hunting method The humpback whale

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observes the position of the small fish and then encircle them. After humpback whales obtain the location of target prey, they will dive into twelve meters down and trail spiral around prey with the bubbles and swim up toward to the surface.

The WOA algorithm is inspired by the special hunting method of humpback whales [16]. Their foraging behavior is called bubble-net attacking method which include following special behavior: encircling prey, spiraling update position, and searching for the prey. The algorithm performs the exploitation phase based on the first two behaviors and the other is the exploration phase. During the search procedure, the whales gradually obtain relevant position of the prey by enclosing and spiraling and capture it finally. The advantages of WOA is simple operation, easy implementation and few adjustment parameters. The WOA have operators to balance between exploration and exploitation that capability to avoid the problem of getting trapped in the local optima.

3.1 Representation and initial population

In solution obtained from WOA is initially represented by vector, the length of which equals the customer. The initial vector of search agent was generated randomly. It must be ensured that there are no repeating number on the same search agent. The example of position of search agent was four search agents as shown in Fig. 1.

The matrix shows the number of search agents in row and the dimensions in the column based on the number of customers. Furthermore, to converting search position to solution representation. We applied the Large Rank Value (LRV) [19] that map continuous numbers into discrete number by ranking the value from largest to smallest, as shown in Fig. 2.

3.2 Encircling Prey

The whales need to decide the position of the prey to surround and capture it when they are foraging. The best search whale assumes that the current optimal candidate solution is the position of the prey or close to the target. The other whales will update their positions according to the optimal candidate solution. This behavior can be described as Eq. (13)-(14).

Whale no.1	0.25	0.42	0.55	0.98	0.88	0.38
Whale no.2	0.13	0.54	0.78	0.16	0.66	0.89
Whale no.3	0.56	0.12	0.67	0.45	0.96	0.24
Whale no.4	0.79	0.43	0.56	0.23	0.11	0.65

Fig. 1 The initialization of positions of the search agent

0.25	0.42	0.55	0.98	0.88	0.38

⇩

6	4	3	1	2	5

Fig. 2 Representation of solution

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$$\vec{D} = \left| \vec{C} \cdot \vec{X}^*(t) - \vec{X}(t) \right| \quad (13)$$

$$\vec{X}(t+1) = \vec{X}^*(t) - \vec{A} \cdot \vec{D} \quad (14)$$

Where t is the current iteration number, $\vec{X}^*$ is the position vector of current optimal solution, $\vec{X}$ is the position vector of the whales, $\vec{D}$ is the distance vector between the whales and current optimal solution, $|\ |$ is the absolute value. $\vec{A}$ and $\vec{C}$ are adjustment vector coefficient that show in Eq. (15)-(16), r is a random number in [0, 1], a is an adjustment parameter whose value decreases linearly from 2 to 0 as the number of iterations increases by Eq. (17).

$$\vec{A} = 2\vec{a} \cdot \vec{r} - \vec{a} \quad (15)$$

$$\vec{C} = 2\vec{r} \quad (16)$$

$$\vec{a} = 2 - t\frac{2}{MaxIter} \quad (17)$$

3.3 Bubble-Net Attacking Method

The WOA algorithm adopted shrinking encirclement and spiraling strategy to update positions of the whales. The shrinking encirclement behavior is realized by adjusting the value of a by Eq. (15). If $|A| < 1$, the whales will approach to the optimal solution from their original position. The position of the whales will be updated by Eq. (14) to realize the shrinking encirclement. In the process of spiraling, a mathematical model is constructed by simulating the spiral behavior of the whales shown as Eq. (18) and (19).

$$\vec{D}^* = \left| \vec{X}^*(t) - \vec{X}(t) \right| \quad (18)$$

$$\vec{X}(t+1) = \vec{D}^* \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) \quad (19)$$

Where D^* is the distance vector between the whales and the prey (current optimal solution), b is a constant coefficient of the spiral shape, l is a random number in [-1, 1].

The whales adopt two strategies of shrinking encirclement and spiral update at the same time to update their locations. The WOA executes these two location update strategies by the value of the probability parameter ρ, where ρ is a random number in [0, 1], which is shown as Eq. (20).

$$\vec{X}(t+1) = \begin{cases} \vec{X}^*(t) - \vec{A} \cdot \vec{D} & if\ p < 0.5 \\ \vec{D}^* \cdot e^{bl} \cdot \cos(2\pi l) + \vec{X}^*(t) & if\ p \geq 0.5 \end{cases} \quad (20)$$

3.4 Searching for Prey

In the process of foraging, the variation of adjustment coefficient A is used to search for prey. If $|A| < 1$, the whales will approach to the prey (exploitation). If $|A| \geq 1$, the whales do not choose the best whale to update their positions but select a random whale to be the best position (exploration). The whales update their positions by Eq. (21) and (22) in the exploration phase. This mechanism of foraging allows the WOA algorithm to perform a global search.

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$$\vec{D} = \left|\vec{C} \cdot \vec{X}_{rand} - \vec{X}\right| \quad (21)$$

$$\vec{X}(t+1) = \vec{X}_{rand} - \vec{A} \cdot \vec{D} \quad (22)$$

Where $\vec{X}_{rand}$ represents the position vector of a whale randomly selected from the current whale population. The pseudo-code of the WOA algorithm is as shown in Fig. 3.

3.5 Local search

In the searching for prey step, we applied the local search to improve solution because the basic WOA has slow convergence speed. The main idea is searching of a defined neighborhood for a candidate solution. It is basically check whether or not there is a better solution in its neighborhood. The local search is divided two types, intra-route searches and inter-route searches. In the intra-route search procedure, the orders of the customers in which serving by the same vehicles, are rearranged if the total distance decreases. On the other hand, in the inter-route local search one or more customers are exchanged between routes if the capacity constraints are not violated and it purposes to reduce the total distance. The 2-Opt operator remove two edges and reconnecting the two paths in different way to obtain a new solution. An example of the 2-Opt as shown in Fig. 4. The single insertion operator, one customer is removed from its current position and random reinsert it at another positions. Like the node insertion,

```
Algorithm 1: Pseudo code of WOA
Initialize the whale's population X_i (i = 1, 2, ..., N)
Calculate the fitness of each search agent, X_i
X* = the best search agent
while (t < MaxIter)
    for each search agent
      Update a, A, c, l and ρ
      if (ρ < 0.5)
         if (|A| < 1)
            Update the position of the current search
            agent by the Eq. (14)
            Appy the inter-route search procedure
         else if (|A| ≥ 1)
            Select a random search agent (X_rand)
            Update the position of the current search
            agent by the Eq. (22)
            Apply the intra-route search procedure
         end if
      else if (ρ ≥ 0.5)
            Update the position of the current search
            by the Eq. (19)
      end if
    end for
    Check if any search agent goes beyond the search
    space and amend it
    Calculate the fitness of each search agent
    Update X* if there is a better solution
    t = t+1
end while
return X*
```

Fig. 3 Pseudo-code of the WOA

this may involve intra route or inter routes. An example of the insertion operator as shown in Fig. 5.

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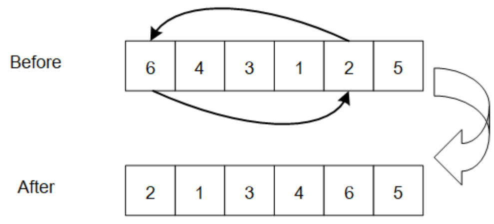

Fig. 4 2-Opt

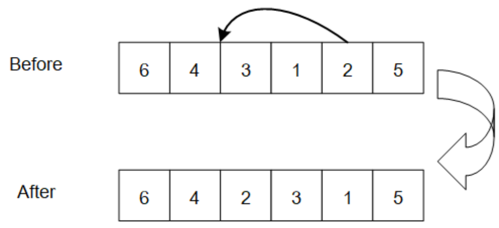

Fig. 5 Single insertion

4. Computational instance

4.1 Problem instance

In this section, we describe the experiment setting for evaluating the quality of the solutions for the proposed approaches discussed previously. The heuristics described in the previous sections is coded in Python. It is estimated by some well-known Solomon's VRPTW instances [24], which are divided into six data sets C1, C2, R1, R2, RC1 and RC2. In sets R1 and R2 customers are uniformly distributed, in sets C1 and C2 customers are clustered in groups and in sets RC1 and RC2 customers are semi-clustered. Each data instance contains the number and capacity of vehicles and the position, demand, time window, service time of all the customers. In addition, there are small time windows and small vehicle capacity in sets C1, R1 and RC1, while there are large time windows and large vehicle capacity in sets C2, R2 and RC2. The problem instance includes 25, 50 and 100 customers in each data set. The 25-customer and 50-customer instance are selected in the first 25 and 50 customers in large problem, respectively.

4.2 Parameters setting

Based on our preliminary evaluation, we define the characteristics of the problems for the experiments. The experiment parameters were search agents (*N*) and the maximum number of iterations (*MaxIter*). The parameter which tuned out to be the most influential on the solution quality. For the parameter setting, *N* is set to 10 in each instance, while *MaxIter* is set to 1,000, 10,000 and 30,000 depend on small to large problem size.

4.3 Result analysis

The result of the proposed algorithm from 10 independent runs is shown Table 1-3. This result is compared with others that are computed by exact method, heuristics, and meta-heuristics. Table 1-2 displays the best-known solution (BKS) and PSO [12] that compared with the best result of WOA. Table 1 shows the WOA provides better solutions 31 out of 56 problems when compared PSO. The WOA has a greater value than PSO in 4 problems of RC instance and 2 problems in R instance (bold and italic).

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Table 1 Computational result of Solomon's 25-customer instances

Problem	BKS	PSO [12]	WOA	Problem	BKS	PSO [12]	WOA
C101	191.3 [25]	191.81	191.81	R112	393 [25]	394.10	394.10
C102	190.3 [25]	190.74	190.74	R201	463.3 [26, 27]	523.66	464.37
C103	190.3 [25]	190.74	190.74	R202	410.5 [26, 27]	455.53	411.49
C104	186.9 [25]	187.45	187.45	R203	391.4 [26, 27]	408.89	394.70
C105	191.3 [25]	191.81	191.81	R204	355 [28, 29]	389.91	355.89
C106	191.3 [25]	191.81	191.81	R205	393 [26, 27]	501.83	395.82
C107	191.3 [25]	191.81	191.81	R206	374.4 [26, 27]	413.21	375.97
C108	191.3 [25]	191.81	191.81	R207	361.6 [27]	402.28	362.63
C109	191.3 [25]	191.81	191.81	R208	328.2 [28, 29]	329.33	329.33
C201	214.7 [26, 30]	215.54	215.54	R209	370.7 [27]	438.24	373.08
C202	214.7 [26, 30]	223.31	215.54	R210	404.6 [26, 27]	513.98	405.48
C203	214.7 [26, 30]	223.31	215.54	R211	350.9 [27]	361.69	352.80
C204	213.1 [26, 27]	221.28	213.93	RC101	461.1 [25]	462.16	***476.96***
C205	214.7 [26, 30]	297.45	215.54	RC102	351.8 [25]	352.74	***401.79***
C206	214.7 [26, 30]	285.39	215.54	RC103	332.8 [25]	333.92	333.92
C207	214.5 [26, 30]	274.78	215.34	RC104	306.6 [25]	307.14	307.14
C208	214.5 [26, 30]	229.84	215.37	RC105	411.3 [25]	412.38	***418.52***
R101	617.1 [25]	618.33	618.33	RC106	345.5 [25]	346.511	***347.31***
R102	547.1 [25]	548.11	548.11	RC107	298.3 [25]	298.95	298.95
R103	454.6 [25]	473.39	464.83	RC108	294.5 [25]	294.99	294.99
R104	416.9 [25]	418.30	417.96	RC201	360.2 [26, 30]	432.30	361.24
R105	530.5 [25]	556.72	531.54	RC202	338.0 [26, 27]	376.12	338.82
R106	465.4 [25]	466.48	***467.85***	RC203	326.9 [28, 29]	432.55	327.69
R107	424.3 [25]	425.27	425.27	RC204	299.7 [29]	327.33	300.23
R108	397.3 [25]	405.39	398.29	RC205	338.0 [27, 29]	386.15	338.93
R109	441.3 [25]	460.52	450.26	RC206	324.0 [27]	482.02	325.10
R110	444.1 [25]	445.80	445.18	RC207	298.3 [27]	478.97	298.95
R111	428.8 [25]	429.70	***431.12***	RC208	269.1 [29]	309.85	269.57
				Total	18551.00	20105.43	18696.64

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Table 2 Computational result of Solomon's 50-customer instances

Problem	BKS	PSO [12]	WOA	Problem	BKS	PSO [12]	WOA
C101	362.4 [25]	363.25	363.25	R112	630.2 [26, 27]	638.49	***677.33***
C102	361.4 [25]	362.17	362.17	R201	791.9 [26, 27	953.29	823.38
C103	361.4 [25]	362.17	***363.34***	R202	698.5 [26, 27]	815.21	739.86
C104	358.0 [25]	358.88	***365.38***	R203	605.3 [28, 29]	668.36	645.19
C105	362.4 [25]	363.25	363.25	R204	506.4 [28]	518.37	***524.75***
C106	362.4 [25]	363.25	363.25	R205	690.1 [28, 29]	756.38	721.25
C107	362.4 [25]	363.25	363.25	R206	632.4 [28, 29]	661.55	***673.14***
C108	362.4 [25]	363.25	***363.92***	R207	-	593.95	***621.07***
C109	362.4 [25]	363.25	363.25	R208	-	508.41	***509.70***
C201	360.2 [26, 30]	441.96	382.12	R209	600.6 [28, 29]	658.28	631.76
C202	360.2 [26, 27]	403.81	377.45	R210	645.4 [28, 29]	670.99	***680.59***
C203	359.8 [26, 27]	402.52	379.14	R211	535.3 [27, 31]	627.40	566.69
C204	350.1 [27]	356.77	***365.54***	RC101	944 [25]	945.58	***966.12***
C205	359.8 [26, 27]	429.12	380.83	RC102	822.5 [25]	823.97	***893.56***
C206	359.8 [26, 27]	412.50	361.41	RC103	710.9 [25]	712.91	***766.70***
C207	359.4 [26, 27]	426.13	370.51	RC104	545.8 [25]	546.51	***561.72***
C208	350.5 [26, 27]	352.29	352.29	RC105	855.3 [25]	856.97	***908.28***
R101	1044.0 [25]	1100.72	1054.88	RC106	723.2 [25]	724.65	***820.47***
R102	909.0 [25]	923.71	***946.67***	RC107	642.7 [25]	645.70	***738.68***
R103	772.9 [25]	790.17	***813.94***	RC108	598.1 [25]	599.70	***600.23***
R104	625.4 [25]	631.58	***663.64***	RC201	684.8 [27, 29]	838.76	690.24
R105	899.3 [25]	983.49	943.95	RC202	613.6 [28, 29]	867.26	622.84
R106	793 [25]	865.93	848.63	RC203	555.3 [28, 29]	674.44	635.42
R107	711.1 [25]	737.10	***763.97***	RC204	444.2 [31]	479.22	461.43
R108	617.7 [26, 27]	624.29	***665.13***	RC205	630.2 [28, 29]	765.02	683.79
R109	786.8 [25]	801.97	***821.64***	RC206	610.0 [28, 29]	755.13	612.65
R110	697.0 [25]	710.40	***725.85***	RC207	558.6 [29]	655.81	570.54
R111	707.2 [26, 27]	756.35	756.21	RC208	-	498.79	496.95
				Total	30953.7	34874.63	34089.19

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Conversely, the proposed algorithm provided an equal total distance at 19 problems. When comparing the results of WOA with BKS in terms of total distance, we found that the performance of our approach is greater than the BKS at 0.72%. Moreover, the proposed method is better than PSO at 6.20%.

Table 2 display the BKS, PSO, and WOA in 50 customers that result of BKS R207, R208, and RC208 not appear in BKS. The WOA can provide a better solution 24 out of 56 problems that compared with PSO. Moreover, the proposed algorithm provided an equal total distance at 7 problems in C instance. The WOA is worse than BKS at 4.48% but it is better than PSO at 2.06%. The result can achieve good solution in C and RC instance. Because the RC2 instance have large time windows and a large vehicle capacity that outperforms PSO.

Table 3 display the best result of WOA that compared with BKS and PSO [12]. The $\%Gap_{BKS}$ and $\%Gap_{PSO}$ were sets with the known optimal solution and PSO result. The $\%Gap$ can be described by the following Eq. (23).

A negative number of *%Gap* shown the proposed algorithm obtained a better value. The numbers in the bold and italic text are the best solutions among proposed algorithms. It can be observed that WOA can obtain the same solutions as BKS in both C1 and C2 instances. The WOA found optimal solution 12 out of 17 problems in C instances. In Table 3 show that the WOA cannot find the equal value or better value in R instances and RC instances. In addition, the average gap of WOA is large than R instances and RC instance which are 7.93% and 8.01%, respectively. The best solution of WOA has a greater value than BKS which is less than 6%. The average gap of the total distance between the BKS and the proposed solutions are within 5.82% of the BKS.

When comparing the results of WOA with PSO in terms of solution quality, the WOA generally provides better solutions for 29 out of 56 instances. We found that the performance of our algorithm is better than the PSO at 1.09% in bold-italic numbers. The performance of the WOA is better than PSO for random problem set and random-cluster problem set in all problems that have large windows and large vehicle capacity.

$$\%\ Gap = \frac{\text{(the WOA solution)} - \text{(the compared solution)}}{\text{the compared solution}} \times 100 \qquad (23)$$

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Table 3 Computational result of Solomon's 100-customer instances

Problem	BKS	PSO [12]	WOA	Gap_{BKS} (%)	Gap_{PSO} (%)
C101	828.94 [32]	828.937	828.94	0.00%	0.00%
C102	828.94 [32]	829.712	828.94	0.00%	***-0.09%***
C103	828.06 [32]	851.373	844.94	2.04%	***-0.76%***
C104	824.78 [32]	868.521	842.16	2.11%	***-3.03%***
C105	828.94 [32]	828.937	828.94	0.00%	0.00%
C106	828.94 [32]	828.937	828.94	0.00%	0.00%
C107	828.94 [32]	828.937	828.94	0.00%	0.00%
C108	828.94 [32]	828.937	828.94	0.00%	0.00%
C109	828.94 [32]	828.937	828.94	0.00%	0.00%
C201	591.56 [32]	591.557	591.56	0.00%	0.00%
C202	591.56 [32]	591.557	609.21	2.98%	2.98%
C203	591.17 [32]	591.173	628.08	6.24%	6.24%
C204	590.60 [32]	615.430	603.40	2.17%	***-1.95%***
C205	588.88 [32]	588.876	588.88	0.00%	0.00%
C206	588.49 [32]	588.876	588.49	0.00%	***-0.07%***
C207	588.29 [32]	591.350	588.29	0.00%	***-0.52%***
C208	588.32 [32]	588.493	588.32	0.00%	***-0.03%***
R101	1642.88 [32]	1652.001	1678.92	2.19%	1.63%
R102	1472.62 [33]	1500.809	1552.26	5.41%	3.43%
R103	1213.62 [33]	1242.649	1315.28	8.38%	5.85%
R104	976.61 [32]	1042.216	1051.54	7.67%	0.89%
R105	1360.78 [32]	1385.082	1475.50	8.43%	6.53%
R106	1240.47 [34]	1294.869	1342.53	8.23%	3.68%
R107	1073.34 [34]	1123.981	1168.01	8.82%	3.92%
R108	947.55 [34]	1011.682	1041.27	9.89%	2.92%
R109	1151.84 [34]	1211.630	1245.09	8.10%	2.76%
R110	1072.41 [32]	1190.362	1153.24	7.54%	***-3.12%***
R111	1053.50 [32]	1102.987	1159.32	10.04%	5.11%

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Table 3 Cont.

Problem	BKS	PSO [12]	WOA	Gap_{BKS} (%)	Gap_{PSO} (%)
R112	953.63 [32]	1029.124	1034.34	8.46%	0.51%
R201	1147.80 [35]	1274.969	1230.86	7.24%	***-3.46%***
R202	1034.35 [32]	1247.033	1134.82	9.71%	***-9.00%***
R203	874.87 [32]	1052.712	948.29	8.39%	***-9.92%***
R204	735.80 [35]	844.161	807.60	9.76%	***-4.33%***
R205	954.16 [35]	1061.460	1036.18	8.60%	***-2.38%***
R206	879.89 [32]	1016.346	944.13	7.30%	***-7.11%***
R207	799.86 [34]	946.778	869.62	8.72%	***-8.15%***
R208	705.45 [36]	834.721	763.69	8.26%	***-8.51%***
R209	859.39 [32]	1003.188	930.16	8.23%	***-7.28%***
R210	910.70 [36]	1040.544	957.24	5.11%	***-8.01%***
R211	755.96 [34]	861.323	815.74	7.91%	***-5.29%***
RC101	1623.58 [37]	1641.204	1732.30	6.70%	5.55%
RC102	1461.23 [32]	1510.952	1598.21	9.37%	5.77%
RC103	1261.67 [38]	1294.739	1395.83	10.63%	7.81%
RC104	1135.48 [39]	1190.545	1239.84	9.19%	4.14%
RC105	1518.58 [32]	1603.707	1651.24	8.74%	2.96%
RC106	1371.69 [40]	1410.931	1479.76	7.88%	4.88%
RC107	1212.83 [34]	1249.795	1297.24	6.96%	3.80%
RC108	1117.53 [33]	1181.870	1239.92	10.95%	4.91%
RC201	1265.56 [32]	1423.519	1326.28	4.80%	***-6.83%***
RC202	1095.64 [32]	1193.591	1184.17	8.08%	***-0.79%***
RC203	928.51 [34]	1123.419	993.68	7.02%	***-11.55%***
RC204	786.38 [35]	894.117	830.08	5.56%	***-7.16%***
RC205	1157.55 [35]	1321.429	1258.78	8.75%	***-4.74%***
RC206	1054.61 [32]	1307.900	1146.50	8.71%	***-12.34%***
RC207	966.08 [32]	1130.368	1060.55	9.78%	***-6.18%***
RC208	779.31 [34]	958.236	818.93	5.08%	***-14.54%***
Total	54728.00	58677.49	58184.83	5.82%	***-1.09%***

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Fig.6 - Fig.8 show the route of optimal solution. Fig. 6 and Fig. 7 show the routing is equal value in BKS, that have 828.94 and 591.56 units in C101 and C201, respectively. On the contrary, the value of C203 cannot find best solution. The WOA provides the large value at 6.24%. It is also an interesting point to observe that customer no. 2, 5, 75, and 93 are served by one other vehicle. Therefore, the C203 instance have 4 vehicles that differ from C201 have 3 vehicles. When we compared the characteristic of instance, the WOA achieved the cluster customer in group.

Fig. 9 shows the best so far solution of each iteration in C1 instances. The convergence rate of WOA is fast in the initial iteration. Most of the instances converge about 25000 iterations that find the objective function at 828.94 unit. However, the convergence rate in C2 is faster than C1 that does not differ in the same group instance at 588.xx units, as shown in Fig. 10.

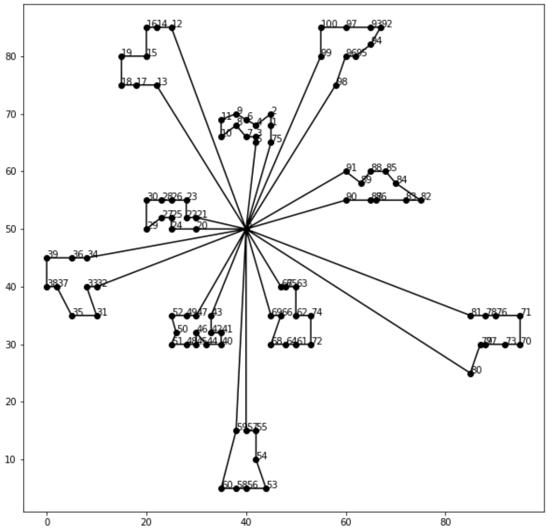

Fig. 6 The route network for instance C101

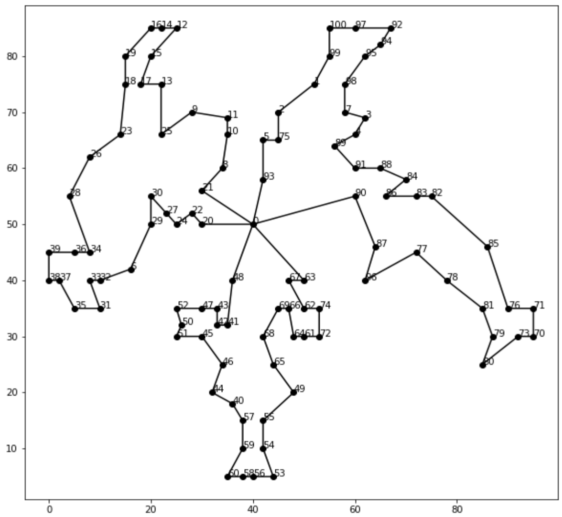

Fig. 7 The route network for instance C201

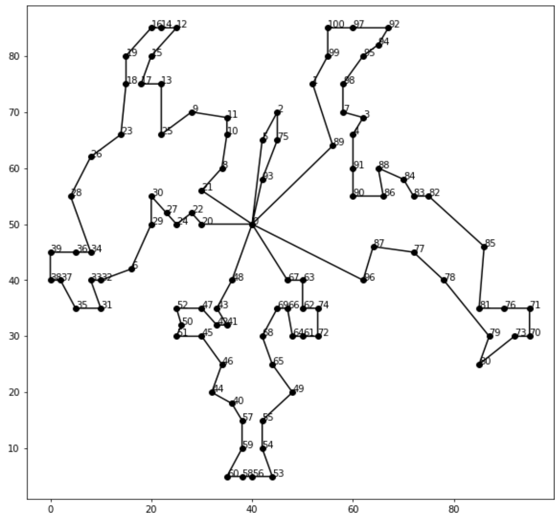

Fig. 8 The route network for instance C203

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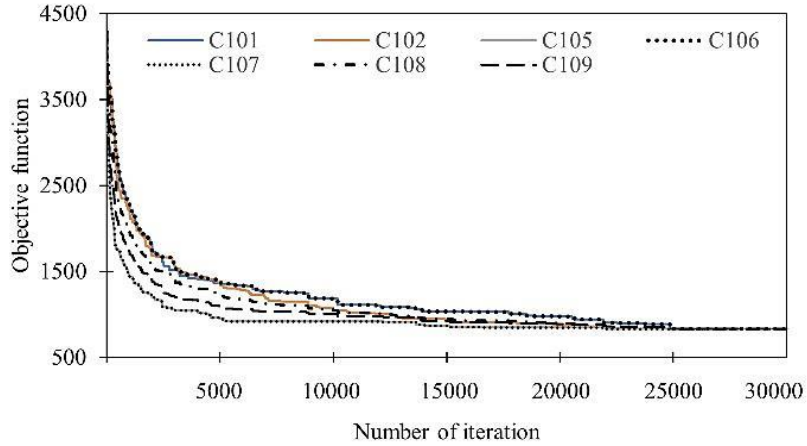

Fig. 9 Convergence graph in C1 instances

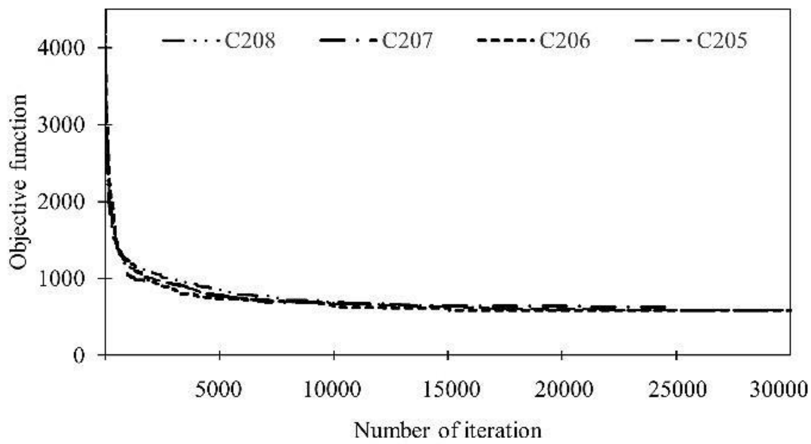

Fig. 10 Convergence graph in C2 instances

5. Conclusion

This paper presented a WOA with local search for solving the VRPTW. The local search is used to improve solution. The approach was tested using problem instances reported in the literature, derived from publicly benchmark data. The experiment results showed that WOA approach was able to find high quality solutions when compared to the benchmark problem for VRPTW. As for future work, it may be interesting to test the proposed WOA with other various VRP such as multi-depot, heterogeneous fleet, dynamic requests, and fuzzy time constraints. However, the effectiveness of WOA needs hybridization.

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