Research Article

Discrete Least Square Estimation of Polynomial Models for ECG Data

Thitanont Charurotkeerati*
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand

Walailuck Chavanasporn
Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand

* Corresponding author. E-mail: wsc@kmutnb.ac.th DOI: 10.14416/j.ijast.2017.11.003
Received: 15 November 2016; Accepted: 3 April 2017; Published online: 24 November 2017

Abstract
The change of the electrical activity of the heart over time has been recorded by the Electrocardiogram which is called ECG. Electrocardiograms are used to diagnose the condition of patients' hearts by measuring the heartbeat. However, an ECG generates a lot of information and requires a large amount of memory and storage to process and record this data. In this paper, a discrete least square estimation is used to estimate the coefficient of polynomial models for estimating the ECG signals. We propose the use of discrete least square estimation in order to fit all of the ECG data, in various orders, with polynomial models using separate parts of the ECG data. A simulation study has been conducted in order to compare the proposed polynomial models with a model restricted by using a Percentage Root mean square Difference (PRD). The results show that the proposed model gives low PRD.

Keywords: ECG data, Discrete least square estimation, Polynomial models

1 Introduction

The Electrocardiogram which is called ECG is used to record the change of the electrical activity of the heart over time. The ECG is a very useful process in medical examination to diagnose the condition of heart muscle in patients during heart beating by looking at both shapes and characteristics.

However, the ECG contains lots of information and therefore it requires large amount of memories storage and time. Several researchers are trying to find ways to make estimating an ECG signal less time consuming see [1], [2] for example. Yim man *et al.* [3] proposed the polynomial model to estimate the ECG signals instead of using the old method, look-up table. They found that the proposed model can produce accurate ECG signals and consume less memory while filling less hardware storage.

Later Desyoo *et al.* [4] proposed to use the discrete least square approximation with kernel function to estimate the ECG signals by fitting the model in to three separated parts. The polynomial model with eight degrees has been used to fit ECG data with minimum PRD. Recently, Charurotkeerati and Chavanasporn [5] also used the discrete least square approximation for fitting the ECG signals by separating the data into three parts. However, they found that the polynomial model with nine degrees was fitted to the ECG data [4]–[9].

In this paper, we extend the work proposed by

Please cite this article as: T. Charurotkeerati and W. Chavanasporn, "Discrete least square estimation of polynomial models for ECG data," *KMUTNB Int J Appl Sci Technol*, vol. 10, no. 4, pp. 257–261, Oct–Dec. 2017.

Charurotkeerati and Chavanasporn [5] comparing their work on three separated parts of data with a model that is restricted to using a Percentage Root mean square Difference (PRD).

2 Materials and Methods

Let y be the vector of ECG data of length N where $\mathbf{y} = (y(1), y(2), ..., y(n))$.

Equation (1) shows the polynomial function $\hat{y}$

$$\hat{y}(n) = a_0 + a_1(n) + a_2(n^2) + ... + a_m(n^m); n = 1, 2, ..., N. \quad (1)$$

Under the normal error assumption, the function $\hat{y}(n)$ is to be chosen to minimize the square error

$$e^2 = \|y(n) - \hat{y}(n)\|^2 \quad (2)$$

The square error in Equation (2) is minimized with the coefficient $a_1, a_2, ..., a_m$ satisfying

$$\frac{\partial e^2}{\partial a_i} = 0; \quad i = 1,2,..., m. \quad (3)$$

where m is a degree of polynomial and m = 9, 19 and 21 respectively.

The coefficients come from solving the system of equations in the Equation (3) which leads to Equation (4).

$$a = (C'C)^{-1} C'\mathbf{y} \quad (4)$$

where

$$C = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 2^m \\ \vdots & \vdots & \ddots & \vdots \\ 1 & N & \cdots & N^m \end{pmatrix}$$

Charurotkeerati and Chavanasporn [5] used simulated ECG data. The data is shown in Figure 1. They fit the polynomial models for one loop of the data using the discrete least square estimation which has 93 data items for each loop as shown in Figure 2. After that they divided the data in to three parts according to the pattern of the ECG signals as shown in Figure 3. They used the polynomial of degree 9 for every part of the data and it gave smaller PRD equal to 2.226%. Figure 4 shows the comparison the original data and its approximation.

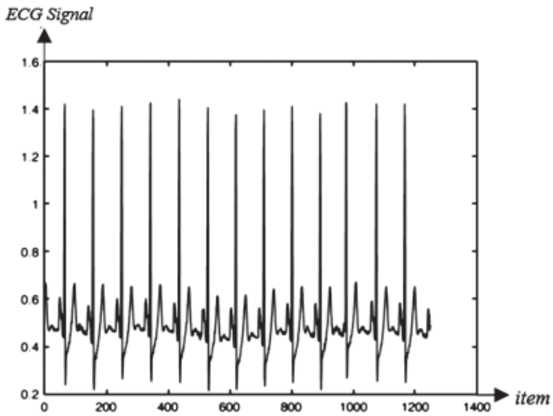

Figure 1: ECG signal simulated from the ECG simulator.

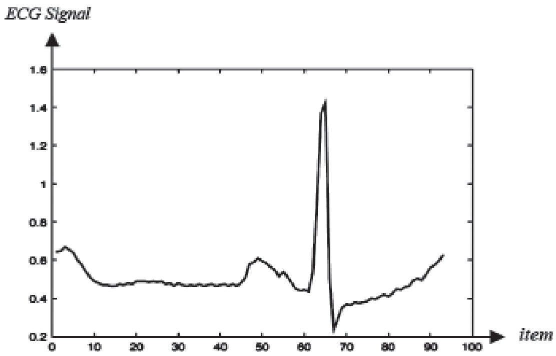

Figure 2: One loop of the ECG data.

The polynomial model of degree 9 for data in part 1 is given as follows:

$$\hat{y} = 0.545260731 + 0.12122192i - 0.033752947i^2 + 0.004567662i^3 - 0.00030361i^4 + 0.0000123i^5 - 0.000000312i^6 + 0.00000000486i^7 - 0.000000000423i^8 + 0.000000000000157i^9 \quad ; \quad i = 1, 2, ..., 60$$

The polynomial model of degree 9 for data in part 2 is given as follows:

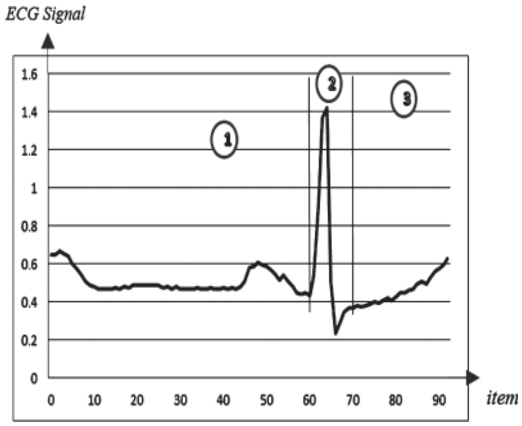

Figure 3: The three parts of one loop of the ECG data.

$$\hat{y} = 0.259048613 + 0.197881407i - 0.12529779i^2 - 130.79521i^3 + 46.1368245i^4 - 10.1672867i^5 + 1.40982986i^6 - 0.11940187i^7 + 0.005638145i^8 - 0.00011369i^9 \;; \quad i = 61, 62, ..., 70$$

And the polynomial model of degree 9 for data in part 3 is given as follows:

$$\hat{y} = 0.2594048613 + 0.197881407i - 0.12529779i^2 + 0.039742916i^3 - 0.00707347i^4 + 0.00075403i^5 - 0.0000491i^6 + 0.00000191i^7 - 0.0000000409i^8 + 0.000000000368i^9 \;; \quad i = 71, 72, ..., 93$$

We propose to adjust the polynomial models proposed by Charurotkeerati and Chavanasporn [5] by fitting the different order for three separated parts of data and comparing it with Charurotkeerati and Chavanasporn [5] using a Percentage Root mean square Difference (PRD). The PRD is given as follows:

$$PRD = \left[\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} y_i^2} \right]^{1/2} \times 100\%$$

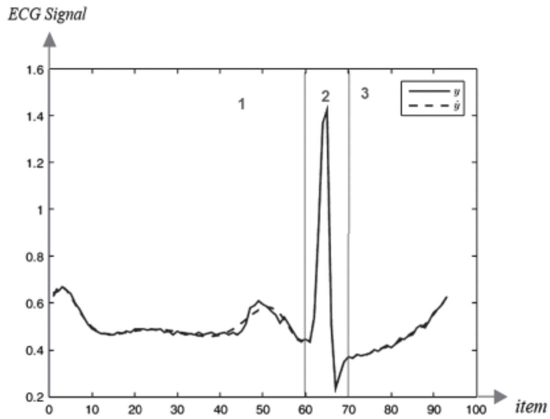

Figure 4: Comparing the original and approximation of ECG data by using polynomial.

3 Results and Discussions

We use same simulated ECG data by Charurotkeerati and Chavanasporn [5] as shown in Figure 1. From Figure 3, we can see that there are three separate patterns of ECG data. We propose to fit the different degree of the polynomial for each part of the data. The discrete least square estimation is used to estimate the coefficients of the ECG model for each part. The minimum PRD for three parts of the simulated ECG data are shown in Table 1 where the 1st part is fitted by the polynomial of degree 21, the 2nd part is fitted by the polynomial of degree 9 and the 3rd part is fitted by the polynomial of degree 19 respectively. The models are given as follows.

The polynomial model of degree 21 for data in part 1 is given as follows:

$$\hat{y} = 0.625562649 + 0.115394969i - 0.22387572i^2 + 0.195819799i^3 - 0.09229i^4 + 0.026991496i^5 - 0.00536055i^6 + 0.000763914i^7 - 8.09406E\text{-}05i^8 + 6.53194\text{E-}06i^9 - 4.08218\text{E-}07i^{10} + 1.99775\text{E-}08i^{11} - 7.70630\text{E-}10i^{12} + 2.34887\text{E-}11i^{13} - 5.64667\text{E-}13i^{14} + 1.06360\text{E-}14i^{15} - 1.54990\text{E-}16i^{16} + 1.71067\text{E-}18i^{17} - 1.38136\text{E-}20i^{18} + 7.69261\text{E-}23i^{19} - 2.63900\text{E-}25i^{20} + 4.20124\text{E-}28i^{21}; \quad i = 1, 2, ..., 60$$

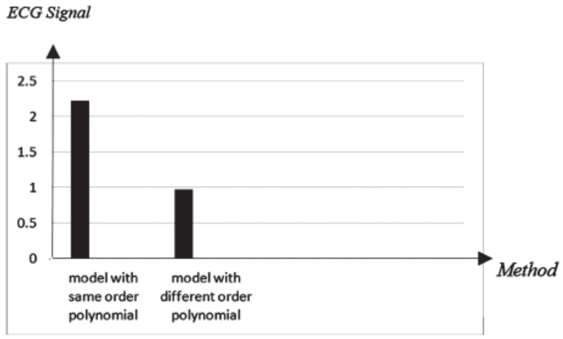

Figure 5: PRD for the polynomial with the same order and the proposed one with a different order.

The polynomial model of degree 9 for data in part 2 is given as follows:

$$\begin{aligned}\hat{y} &= 73.255 - 200.82048i + 221.53021i^2 \\ &- 130.79521i^3 + 46.136825i^4 - 10.167287i^5 \\ &+ 1.4098299i^6 - 0.1194019i^7 + 0.0056381i^8 \\ &- 0.0001137i^9; \quad i = 61, 62, \ldots, 70\end{aligned}$$

And the polynomial model of degree 19 for data in part 3 is given as follows:

$$\begin{aligned}\hat{y} &= -282.4551226 + 943.6641963i - 1354.290275i^2 \\ &+ 1129.363233i^3 - 620.5382751i^4 + 240.7435189i^5 \\ &- 68.85114856i^6 + 14.93625958i^7 - 2.505694169i^8 \\ &+ 0.329225587i^9 - 0.034135684i^{10} + 0.002801328i^{11} \\ &- 0.000181678i^{12} + 9.25149\text{E-}06i^{13} - 3.65260\text{E-}07i^{14} \\ &+ 1.09458\text{E-}08i^{15} - 2.40469\text{E-}10i^{16} + 3.65036\text{E-}12i^{17} \\ &- 3.41977\text{E-}14i^{18} + 1.48921\text{E-}16i^{19}\ ; i = 71, 72, \ldots, 93\end{aligned}$$

Table 1: The minimum PRD for each part for simulated ECG data

Part	PRD
1	1.2317
2	4.9542e-9
3	0.5046
Over all	0.9730

The overall PRD for the proposed polynomial model for ECG data is 0.973 where it is smaller than the model proposed by Charurotkeerati and Chavanasporn [5]. The comparison of the PRD for the proposed model and the Charurotkeerati and Chavanasporn [5] model is shown in Figure 5.

In Figure 5, we can see that the proposed polynomial model with the different order of polynomial for each part of the data performs better than the Charurotkeerati and Chavanasporn [7] model with the same order of polynomial.

4 Conclusions

In this paper, we propose to adjust the polynomial models proposed by Charurotkeerati and Chavanasporn [5] by fitting the different degrees of polynomial for three separated parts of data and comparing it with Charurotkeerati and Chavanasporn [5] using a percentage root mean square difference. We can see that the proposed polynomial model with the different order of polynomial for each part of the data performs better than the Charurotkeerati and Chavanasporn [5] model with the same order of polynomial. Therefore, it is a very useful technique that can be used to make high resolution ECG simulation less time consuming. The work carried out in this research has revealed many promising areas of further research in design polynomial models.

References

[1] B. Madhukar and I. S. N. Murthy, "ECG data compression by modeling," in *Proceedings of the Annual Symposium on Computer Application in Medical Care*, 1992, pp. 586–591.

[2] D. Haugland, J. Heber, and J. Husoy, "Optimaisation algorithm for ECG data compression," *Medical and Biological Engineering and Computing*, vol. 35, no. 4, pp.420–424, Jul. 1997.

[3] S. Yimman and M. Deeudom, "An implementation of discrete mathematical model for ECG waveform," in *Proceeding for International Conference on Control, Automation, and Systems (ICCAS)*, 2005.

[4] P. Desyoo, S. Praesomboon, W. Sangpetch, and W. Surachertdkiati, "Discrete mathematical

model for ECG waveform using kernel function," in *Proceedings ICROS-SICE International Joint Conference*, 2009, pp. 5296–5300.

[5] T. Charurotkeerati and W. Chavanasporn, "Polynomial approximation for ECG data," in *Proceedings of the 5th National and International Graduate Study Conference*, 2015, pp. 795–800.

[6] M.H. Hayes, *Statistical Digital Signal Processing and Modeling*. New York: John Wiley & Sons, inc., 1996.

[7] M. Karczewica and M. Gabbouj, "ECG data compression by spline approximation," *Signal Processing*, vol. 59, pp.43–59, 1997.

[8] R. Nygaard and D. Haugland, "Compressing ECG signals by piecewise polynomial approximation," in *Proceedings of the 1998 IEEE International Conference on Acoustics, Speech and Signal Processing*, 1998, pp. 1809–1812.

[9] W. Philips and G. D. Jonghe, "Data compression of ECG's by high-degree polynomial approximation, IEEE *Transactions on Biomedical Engineering*, vol. 39, pp.330–337, 1992.