Research Article

Performance Measurement of a DMEWMA Control Chart on an AR(p) Model with Exponential White Noise

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Abstract
The double-modified exponentially weighted moving average (DMEWMA) control chart running an autoregressive (AR) process is proposed to detect unusual events. The AR equation and the DMEWMA statistic are combined to evaluate the control limit of the exponential residual term to obtain the explicit formula for the average run length (ARL). The ARLs computed using the explicit formula approach and the well-established numerical integral equation method were compared to validate the former. The efficiencies of the original EWMA, MEWMA, and DMEWMA control charts running AR processes based on simulation and real data were compared by using the results of ARL and relative mean index calculations. The results indicate that the explicit formula for the ARL of an AR process running on a double-modified EWMA control chart detected changes more quickly than on either of the other two control charts for small and moderate changes. Finally, real data on COVID-19 is provided to demonstrate the application of this explicit formula.

Keywords: Explicit formula, Average run length, Double modified exponentially weighted moving average, Autoregressive model, COVID-19

1 Introduction

Control charts comprise an important instrument for statistical process control (SPC) and are extensively used for monitoring processes. The Shewhart control chart was the first to be reported and is capable of detecting large shifts in a process parameter [1]. Subsequently, the cumulative sum (CUSUM) [2] and exponentially weighted moving average (EWMA) [3] control charts were proposed as alternative tools capable of detecting small changes in a process parameter and dealing with autocorrelated observations. The EWMA scheme has been used in several studies. Moving-average and EWMA control charts have also been used to explain the number of COVID-19 cases in Karkh General Hospital, Iraq [4]. Yupaporn and Rapin [5] determined the alert level for the number of new COVID-19 cases in Thailand, Singapore, Vietnam, and Hong Kong by using a quantile function with the EWMA control chart. In India, COVID-19 was monitored using the EWMA control chart to determine when new COVID-19 cases and deaths increased speedily [6].

Later, Khan *et al.*, [7] improved the original EWMA statistic to provide the modified EWMA (MEWMA) control chart. Its capability has been investigated using various real phenomena such as air pollution data [8], COVID-19 cases [9], and cancer [10]. Recently, Alevizakos *et al.*, [11] proposed the double-modified EWMA (DMEWMA) control chart for monitoring shifts in the mean of a normally distributed process and applied it to an industrial process.

In a surveillance system, the observations are time-dependent and often consist of autocorrelated sequences that require time series models to solve their associated problems. Using control charts and time series models together is required for accurate analysis of disease data. Many studies have exploited time series models for predicting COVID-19 data [12]–[14].

One measure to evaluate the performance of a control chart is the average run length (ARL), which can be found as an exact solution using an explicit formula. For an EWMA scheme of an autoregressive (AR) process with exponential white noise, the explicit formulas for the ARL were derived [15]. Afterward, the ARL solutions from the explicit formula were presented to test the efficiency of a MEWMA chart on an AR model with exponential white noise [16]. Next, the derivation of the explicit formulas for ARL on an extended EWMA (EEWMA) control chart of an AR process with exponential white noise was proposed to measure the performance of this control chart [17]. Recently, Silpakob *et al.*, [18] developed the explicit formulas of the ARL for an AR model with exponential white noise on a new MEWMA scheme.

Since the advent of COVID-19 in 2019 [19], many countries now realize the danger of a new pandemic. The daily cases of COVID-19 have been reported continuously. These important phenomena can be monitored by using SPC as a control chart.

In this study, the explicit formula was derived for the ARL of an autoregressive (AR) process with exponential white noise running on a DMEWMA control chart. This ARL approach was validated against the estimated ARL using the numerical integral equation (NIE) method [20]. After that, the efficacies of EWMA, MEWMA, and DMEWMA control charts using simulated data and datasets of COVID-19 cases in various countries were compared using the proposed method.

2 Materials and Methods

2.1 *Statistical measurements*

The DMEWMA control chart is an enhanced version of the original EWMA and MEWMA schemes that can more rapidly detect changes in autocorrelated or independent normally distributed observations. The DMEWMA statistic can be defined as Equation (1).

$$D_t = (1-\lambda_2)D_{t-1} + \lambda_2 M_t + c_2(M_t - M_{t-1}), \quad (1)$$

where $M_t = (1-\lambda_1)M_{t-1} + \lambda_1 A_t + c_1(A_t - A_{t-1})$ is the MEWMA statistic, λ_1 and λ_2 are positive exponential smoothing parameters (λ_1, λ_2 < 1), c_1 and c_2 are suitable constants, and A_t is a data sequence at t = 1, 2, 3,... with mean μ and standard deviation σ. The lower and upper control limits [LCL, UCL] can be written as Equation (2).

$$\mu \pm W_m \sigma \sqrt{C_\sigma}, \quad (2)$$

where W_m comprises suitable control width limits and C_σ is a standard deviation constant with $c = c_1 = c_2$, $\lambda = \lambda_1 = \lambda_2$ and $\theta = (1-\lambda)^2$ defined as

$$\begin{aligned} C_\sigma = {} & (c+\lambda)^4 + 4\lambda^2(c+\lambda)^2(c+\lambda-1)^2 \\ & + \frac{4\lambda^2(c+\lambda)^2(c+\lambda-1)^2(1-\lambda)^2}{1-\theta} \\ & - \frac{4\lambda^3(c+\lambda)(c+\lambda-1)^3(1-\lambda)}{(1-\theta)^2} \\ & + \lambda^4(c+\lambda-1)^4\frac{(1+\theta)}{(1-\theta)^3}. \end{aligned}$$

If a dataset can be suitably modeled as AR processes with exponential white noise, then the sequences of A_t in an AR(p) model can be written as Equation (3).

$$A_t = \phi_0 + \sum_{i=1}^{p}\phi_i A_{t-i} + \varepsilon_t, \quad (3)$$

where ϕ_0 is the process mean, ϕ_i (i = 1,2,…,p) is the coefficient of the AR model ($-1 < \phi_i < 1$) and ε_t is the exponential white noise sequence of independent random variables as $\varepsilon_t \sim Exp(\beta)$.

Therefore, the DMEWMA statistic for an AR(p) model is given by

$$\begin{aligned} D_t = {} & (1-\lambda_2)D_{t-1} + [(1-\lambda_1)\lambda_2 + (1-\lambda_1)c_2 - c_2]M_{t-1} \\ & + (\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)(\phi_0 + \varepsilon_t) \\ & + (\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)\sum_{i=1}^{p}\phi_i A_{t-i} - (c_1\lambda_2 + c_1c_2)A_{t-1}. \end{aligned}$$

Since the interval at D_1 in the control process is $[L, U]$ to determine the initial value $D_0 = s$, we can obtain

$$\begin{aligned}\psi = &(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)\phi_0 \\ &+(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)\sum_{i=1}^{p}\phi_i A_{1-i} - (c_1\lambda_2 + c_1c_2)A_0 \\ &+[(1-\lambda_1)\lambda_2 + (1-\lambda_1)c_2 - c_2]\times \\ &[(1-\lambda_1)M_{-1} + \lambda_1 A_0 + c_1(A_0 - A_{-1})],\end{aligned}$$

then

$$L \le (1-\lambda_2)s + (\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)\varepsilon_1 + \psi \le U.$$

After that, the control limit is transferred to the exponential residual term ε_t, as follows:

$$\frac{L-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)} \le \varepsilon_1 \le \frac{U-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}$$

such that

$$P(LCL \le \varepsilon_1 \le UCL) = \int_{\frac{L-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}}^{\frac{U-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}} f(\varepsilon_1)d\varepsilon_1.$$

2.2 *Method of evaluating control chart*

The efficiency of a control chart can be evaluated by using the ARL, which can be solved using the Fredholm integral equation of the second kind [21]. To detect a shift in the process mean, the control chart performs best when the ARL for a particular scenario is the lowest. For the DMEWMA statistic and an AR(p) model with exponential white noise, the ARL denoted $Y(s)$ can be written as

$$P(LCL \le \varepsilon_1 \le UCL) = \int_{\frac{L-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}}^{\frac{U-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}} f(\varepsilon_1)d\varepsilon_1.$$

$$Y(s) = 1 + \frac{1}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}\times \int_l^u Y(z) f\left[\frac{z-(1-\lambda_2)s-\psi}{(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}\right]dz. \quad (4)$$

The explicit formula for the ARL for the DMEWMA statistic and an AR(p) model can be derived by solving the integral equation of $Y(s)$ in Equation (4) with an exponential function as follows:

$$Y(s) = 1 + \frac{e^{\frac{(1-\lambda_2)s+\psi}{\beta(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}}}{\beta(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}\times \int_l^u e^{\frac{-z}{\beta(\lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2)}} Y(z)dz.$$

If $\gamma = \lambda_1\lambda_2 + c_1\lambda_2 + c_2\lambda_1 + c_1c_2$ and $B = \int_l^u e^{\frac{-z}{\beta\gamma}} Y(z)dz$, then $Y(s) = 1 + \frac{e^{\frac{(1-\lambda_2)s+\psi}{\beta\gamma}}}{\beta\gamma}\times B$ Moreover, B can be solved as follows:

$$B = \int_l^u e^{\frac{-z}{\beta\gamma}}\left(1 + \frac{e^{\frac{(1-\lambda_2)z+\psi}{\beta\gamma}}}{\beta\gamma}\cdot B\right)dz$$

$$B = -\beta\gamma(e^{\frac{-u}{\beta\gamma}} - e^{\frac{-l}{\beta\gamma}}) - \frac{e^{\frac{\psi}{\beta\gamma}}}{\lambda_2}(e^{\frac{-u\lambda_2}{\beta\gamma}} - e^{\frac{-l\lambda_2}{\beta\gamma}})B$$

$$B = \frac{-\beta\gamma(e^{\frac{-u}{\beta\gamma}} - e^{\frac{-l}{\beta\gamma}})}{1 + \frac{e^{\frac{\psi}{\beta\gamma}}}{\lambda_2}(e^{\frac{-u\lambda_2}{\beta\gamma}} - e^{\frac{-l\lambda_2}{\beta\gamma}})}.$$

Afterward,

$$Y(s) = 1 + \frac{e^{\frac{(1-\lambda_2)s+\psi}{\beta\gamma}}}{\beta\gamma}\left(\frac{-\beta\gamma(e^{\frac{-u}{\beta\gamma}} - e^{\frac{-l}{\beta\gamma}})}{1 + \frac{e^{\frac{\psi}{\beta\gamma}}}{\lambda_2}(e^{\frac{-u\lambda_2}{\beta\gamma}} - e^{\frac{-l\lambda_2}{\beta\gamma}})}\right)$$

$$Y(s) = 1 - \frac{\lambda_2 e^{\frac{(1-\lambda_2)s}{\beta\gamma}}(e^{\frac{-u}{\beta\gamma}} - e^{\frac{-l}{\beta\gamma}})}{\lambda_2 e^{\frac{-\psi}{\beta\gamma}} + e^{\frac{-u\lambda_2}{\beta\gamma}} - e^{\frac{-l\lambda_2}{\beta\gamma}}}.$$

$$Y(s) = 1 - \frac{\lambda_2 e^{\frac{(1-\lambda_2)s}{\beta\gamma}}(e^{\frac{-u}{\beta\gamma}} - e^{\frac{-l}{\beta\gamma}})}{\lambda_2 e^{\frac{-\psi}{\beta\gamma}} + e^{\frac{-u\lambda_2}{\beta\gamma}} - e^{\frac{-l\lambda_2}{\beta\gamma}}}. \quad (5)$$

Therefore, $Y(s)$ in Equation (5) is the explicit formula for the ARL of the DMEWMA statistic and an AR(p) model.

The ARL estimated via the NIE method [20] derived by using the composite midpoint quadrature rule denoted $Y_N(s)$ is a well-known technique that can be used to verify the ARL via the explicit formula. This rule gives ARL values close to other techniques and the lowest CPU time. $Y_N(s)$ for an AR(p) model on the DMEWMA control chart can be determined via the k linear equation system on the interval $[l, u]$, where the length of k is equally divided into intervals, i.e., $h_j = \frac{u-l}{k}$ with the middle point of the j^{th} interval $z_j = (j - \frac{1}{2})h_j + l$ From Equation (4), $Y_N(s)$ can be defined as Equation (6).

$$Y_N(s) \approx 1 + \frac{1}{\gamma}\sum_{j=1}^{k} h_j \times Y(z_j) f\left[\frac{z_j - (1-\lambda_2)s - \psi}{\gamma}\right] \quad (6)$$

3 Results and Discussion

3.1 *The accuracy of the proposed method*

For a practice of an AR process with exponential white noise running on a DMEWMA chart, the initial ARL is determined as 370 on control limit $[l, u]$ with $\beta = 1$ in an exponential distribution such that the lower bound (l) is set to be a constant for finding the upper bound (u). The NIE method is calculated for k linear equations as 1000. After that, the process mean is examined for various shift sizes (δ) = 0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, and 1 for the out-of-control process. The ARL difference between the proposed explicit formula and the NIE method is conducted to determine the precision of the former. The relative percentage change (RPC) [22] is brought to compare solutions of two techniques as follows in Equation (7):

$$RPC = \left|\frac{Y(s) - Y_N(s)}{Y(s)}\right| \times 100\%. \quad (7)$$

Tables 1–3 show the ARL results using the explicit formula ($Y(s)$) and the NIE ($Y_N(s)$) method for AR(1), AR(2) and AR(3) processes ($\phi_0 = 1$, $\phi_1 = 0.05$, $\phi_2 = 0.1$, $\phi_3 = 0.15$) running on a DMEWMA control chart with adjusted c_1, c_2, λ_1 and λ_2. It can be seen that all RPC results are very low. Thus, this explicit formula approach can be confidently used to calculate the ARL of AR(p) processes running in a DMEWMA control chart.

The ARL and the relative mean index (RMI) [16] are used to compare the performances of the control charts. The best-performing control chart for a particular scenario provides the lowest ARL and RMI values. The RMI is computed as Equation (8).

$$RMI = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{ARL_i(r) - ARL_i(m)}{ARL_i(m)}\right], \quad (8)$$

where $ARL_i(r)$ is the ARL of row i on the evaluated control chart and $ARL_i(m)$ is the lowest ARL for row i of all of the compared control charts.

Table 1: The ARLs of the explicit formula and the NIE method running on a DMEWMA control chart of an AR(p) model with control limit $[0.9, u]$ at $c_1 = 0.5$, $c_2 = 0.5$ and $\lambda_1 = 0.05$, $\lambda_2 = 0.05$

δ	AR(1) u = 1.189139557			AR(2) u = 1.160996924			AR(3) u = 1.123930457		
	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC
0.000	370.000221	370.000192	7.8×10^{-6}	370.000205	370.000182	6.2×10^{-6}	370.000377	370.000360	4.6×10^{-6}
0.001	259.549270	259.549253	6.5×10^{-6}	255.125759	255.125746	5.1×10^{-6}	249.062215	249.062206	3.6×10^{-6}
0.002	199.993279	199.993267	6×10^{-6}	194.797286	194.797277	4.6×10^{-6}	187.825832	187.825826	3.2×10^{-6}
0.005	118.687392	118.687386	5.1×10^{-6}	114.191704	114.191699	4.4×10^{-6}	108.328052	108.328049	2.8×10^{-6}
0.01	71.0257957	71.0257926	4.4×10^{-6}	67.8558787	67.8558763	3.5×10^{-6}	63.7859567	63.7859551	2.5×10^{-6}
0.02	39.7053871	39.7053855	4×10^{-6}	37.7663321	37.7663308	3.4×10^{-6}	35.2996849	35.2996841	2.3×10^{-6}
0.05	17.5515075	17.5515069	3.4×10^{-6}	16.6531709	16.6531704	3×10^{-6}	15.5144668	15.5144664	2.6×10^{-6}
0.10	9.49048109	9.49048082	2.8×10^{-6}	9.00621585	9.00621565	2.3×10^{-6}	8.39058455	8.39058441	1.7×10^{-6}
0.20	5.31840971	5.31840960	2.1×10^{-6}	5.05706647	5.05706638	1.8×10^{-6}	4.72241142	4.72241136	1.3×10^{-6}
0.50	2.76357879	2.76357876	1.1×10^{-6}	2.64457090	2.64457088	0.8×10^{-6}	2.49009673	2.49009672	4×10^{-6}
1.00	1.89796984	1.89796983	0.5×10^{-6}	1.83104603	1.83104603	0	1.74339634	1.74339633	0.6×10^{-6}

Table 2: The ARLs of the explicit formula and the NIE method running on a DMEWMA control chart of an AR(p) model with control limit [0.9, u] at $c_1 = 0.5$, $c_2 = 0.5$ and $\lambda_1 = 0.05$, $\lambda_2 = 0.10$

δ	AR(1) u = 1.218715682			AR(2) u = 1.186999265			AR(3) u = 1.1454635061		
	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC
0.000	370.000138	370.000063	2.0×10^{-5}	370.000353	370.000295	1.6×10^{-5}	370.000047	370.000006	1.1×10^{-5}
0.001	258.646860	258.646821	1.5×10^{-5}	253.875563	253.875533	1.2×10^{-5}	247.372061	247.372041	8.1×10^{-6}
0.002	198.928238	198.928213	1.3×10^{-5}	193.349040	193.349021	9.8×10^{-6}	185.917375	185.917362	7×10^{-6}
0.005	117.763786	117.763775	9.3×10^{-6}	112.966310	112.966302	7.1×10^{-6}	106.765965	106.765960	4.7×10^{-6}
0.01	70.3789688	70.3789641	6.7×10^{-6}	67.0089284	67.0089249	5.2×10^{-6}	62.7256686	62.7256663	3.7×10^{-6}
0.02	39.3179828	39.3179808	5.1×10^{-6}	37.2622582	37.2622566	4.3×10^{-6}	34.6750936	34.6750926	2.9×10^{-6}
0.05	17.3843125	17.3843118	4×10^{-6}	16.4348274	16.4348269	3×10^{-6}	15.2445603	15.2445599	2.6×10^{-6}
0.10	9.41046013	9.41045983	3.2×10^{-6}	8.90000196	8.90000174	2.5×10^{-6}	8.25825110	8.25825095	1.8×10^{-6}
0.20	5.28418901	5.28418889	2.3×10^{-6}	5.00971540	5.00971531	1.8×10^{-6}	4.66206194	4.66206188	1.3×10^{-6}
0.50	2.75583954	2.75583951	1.1×10^{-6}	2.63162029	2.63162027	0.8×10^{-6}	2.47199534	2.47199533	0.4×10^{-6}
1.00	1.89728792	1.89728792	0	1.82774053	1.82774053	0	1.73746918	1.73746917	0.6×10^{-6}

Table 3: The ARLs of the explicit formula and the NIE method running on a DMEWMA control chart of an AR(p) model with control limit [0.9, u] at $c_1 = 0.5$, $c_2 = 1$ and $\lambda_1 = 0.05$, $\lambda_2 = 0.10$

δ	AR(1) u = 1.456659474			AR(2) u = 1.40247499			AR(3) u = 1.331110598		
	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC	$Y(s)$	$Y_N(s)$	RPC
0.000	370.000207	370.000176	8.4×10^{-6}	370.000200	370.000175	6.8×10^{-6}	370.000081	370.000063	4.9×10^{-6}
0.001	248.077803	248.077786	6.9×10^{-6}	243.382572	243.382559	5.3×10^{-6}	236.982008	236.981999	3.8×10^{-6}
0.002	186.735058	186.735047	5.9×10^{-6}	181.476438	181.476429	5×10^{-6}	174.464859	174.464853	3.4×10^{-6}
0.005	107.488568	107.488562	5.6×10^{-6}	103.213691	103.213687	3.9×10^{-6}	97.6691335	97.6691307	2.9×10^{-6}
0.01	63.2779149	63.2779121	4.4×10^{-6}	60.3669723	60.3669701	3.6×10^{-6}	56.6461319	56.6461305	2.5×10^{-6}
0.02	35.0741388	35.0741374	4×10^{-6}	33.3314387	33.3314377	3×10^{-6}	31.1221043	31.1221035	2.6×10^{-6}
0.05	15.5143812	15.5143807	3.2×10^{-6}	14.7166899	14.7166895	2.7×10^{-6}	13.7080715	13.7080713	1.5×10^{-6}
0.10	8.47269400	8.47269375	3×10^{-6}	8.04255104	8.04255086	2.2×10^{-6}	7.49683646	7.49683634	1.6×10^{-6}
0.20	4.83907440	4.83907430	2.1×10^{-6}	4.60513721	4.60513717	0.9×10^{-6}	4.30607223	4.30607217	1.4×10^{-6}
0.50	2.60675083	2.60675080	1.2×10^{-6}	2.49784810	2.49784808	0.8×10^{-6}	2.35665115	2.35665114	0.4×10^{-6}
1.00	1.83897667	1.83897666	0.5×10^{-6}	1.77627248	1.77627248	0	1.69421066	1.69421066	0

Tables 4–6 show the performance comparison of EWMA, MEWMA and DMEWMA control charts with vary c_1, c_2 using the ARL at λ_1, $\lambda_2 = 0.05$ for AR(1), AR(2) and AR(3) models, respectively. These ARL results show in the same direction such that the ability of the DMEWMA control chart is better for the level of small and medium changes while the EWMA scheme is more capable in a large shift size. Moreover, the RMI values are calculated for comparing the summarized results of each control chart in Table 7 such that the RMI values of the DMEWMA control chart are lower than the EMMA and MEWMA at the same c_1 and c_2. Moreover, the DMEWMA control charts with large c_1 and c_2 values give lower ARL and RMI values and more capability.

3.2 *Application to real data*

Confirmed cases of COVID-19 data [19] in several countries were used in this study. All datasets comprised 120 observations of the daily confirmed cases of COVID-19 in Malaysia, Japan and Thailand from 1 January 2022 to 30 April 2022. These datasets were evaluated for fit to the AR(p) model by using the t-statistic test from the Box-Jenkins method and they were tested for exponential white noise by using the Kolmogorov-Smirnov statistic. The case of Malaysian COVID-19 data was forecasted to be suitable for an AR(1) model as $A_t = 0.989A_{t-1} + \varepsilon_t$ and $\varepsilon_t \sim Exp(1527.08)$ Similarly, the COVID-19 case data of Japan was predicted to fit an AR(2) model as established via $A_t = 0.539A_{t-1} + 0.434A_{t-2} + \varepsilon_t$ and $\varepsilon_t \sim Exp(9002.41)$. For Thailand, this COVID-19 dataset was tested as acceptable in an AR(3) model as follows $A_t = 1.287A_{t-1} - 0.543A_{t-2} + 0.251A_{t-3} + \varepsilon_t$ and $\varepsilon_t \sim Exp(1191.81)$

In Tables 8–10, the ARL results at $\lambda_1, \lambda_2 = 0.05$ for EWMA, MEWMA and DMEWMA control charts for various δ, c_1 and c_2 values are shown with an AR(1) model representing the Malaysian COVID-19 data, AR(2) model representing the COVID-19 case data of Japan and an AR(3) model representing the COVID-19 data in Thailand, respectively, with the corresponding RMI results provided in Table 11. It can be seen that the DMEWMA control chart with large c_1 and c_2 values shows lower ARL and RMI values than the EWMA and MEWMA control charts, thereby indicating its better applicability for all processes.

Table 4: The ARLs for the simulated data fitted to an AR(1) model on the various control charts with control limit [1, *u*]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
	u = 1.0486412	**1.535053**	**2.021466**	**2.99429**	**10.7769**	**1.29428**	**2.07254**	**3.094**	**5.0883**	**21.0426**	**99.2577**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	239.001	179.808	177.769	176.647	175.805	183.666	177.691	176.494	176.121	175.584	175.529
0.002	176.705	119.025	117.230	116.270	115.515	122.348	117.150	116.180	115.790	115.369	115.306
0.005	99.487	59.4545	58.3478	57.7661	57.2975	61.4854	58.2938	57.7270	57.4654	57.2250	57.1819
0.01	57.9209	32.7680	32.1120	31.7695	31.4915	33.9694	32.0792	31.7495	31.5905	31.4519	31.4255
0.02	31.9456	17.6261	17.2648	17.0768	16.9237	18.2872	17.2465	17.0667	16.9781	16.9028	16.8880
0.05	14.1711	7.86854	7.71219	7.63099	7.56467	8.15447	7.70422	7.62683	7.5882	7.5559	7.54943
0.10	7.81705	4.49285	4.41021	4.36731	4.33224	4.64387	4.40599	4.36515	4.34467	4.32764	4.32421
0.20	4.54345	2.77817	2.73369	2.71060	2.69171	2.85935	2.73142	2.70945	2.69840	2.68924	2.68739
0.50	2.52419	1.73440	1.71384	1.70315	1.69441	1.77187	1.71278	1.70262	1.69751	1.69327	1.69241
1.00	1.81832	1.37833	1.36653	1.36039	1.35537	1.39982	1.36592	1.36009	1.35715	1.35471	1.35422

Table 5: The ARLs for the simulated data fitted to an AR(2) model on the various control charts with control limit [1, *u*]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
u	**1.0439061**	**1.482967**	**1.92203**	**2.80015**	**9.82513**	**1.265632**	**1.96813**	**2.89016**	**4.69031**	**19.0915**	**89.6926**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	241.959	183.172	181.185	179.983	179.094	186.911	181.005	179.989	179.426	178.933	178.907
0.002	179.948	121.969	120.176	119.168	118.384	125.278	120.049	119.147	118.669	118.262	118.212
0.005	102.045	61.2640	60.1365	59.5321	59.0473	63.3294	60.0702	59.5093	59.2210	58.9796	58.9384
0.01	59.6262	33.8339	33.1605	32.8054	32.5178	35.0661	33.1235	32.7899	32.6202	32.4790	32.4527
0.02	32.9437	18.2048	17.8324	17.6377	17.4792	18.8858	17.8127	17.6287	17.5356	17.4583	17.4432
0.05	14.6076	8.10819	7.94675	7.86274	7.79416	8.40333	7.93836	7.85869	7.81848	7.78518	7.77854
0.10	8.03767	4.61084	4.52555	4.48123	4.44500	4.76667	4.52115	4.47906	4.45784	4.44028	4.43674
0.20	4.65028	2.83389	2.78808	2.76428	2.74481	2.91751	2.78572	2.76311	2.75171	2.74228	2.74037
0.50	2.56152	1.75306	1.73199	1.72105	1.71208	1.79145	1.73091	1.72051	1.71526	1.71092	1.71004
1.00	1.83333	1.38553	1.37351	1.36726	1.36214	1.40741	1.37289	1.36690	1.36396	1.36148	1.36098

Table 6: The ARLs for the simulated data fitted to an AR(3) model on the various control charts with control limit [1, u]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
u	**1.0376699**	**1.414369**	**1.791068**	**2.544465**	**8.57165**	**1.227903**	**1.83062**	**2.62169**	**4.16615**	**16.5219**	**77.095**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	245.994	187.773	185.676	184.540	183.656	191.490	185.489	184.535	183.905	183.589	183.397
0.002	184.378	126.022	124.152	123.157	122.363	129.373	124.020	123.131	122.616	122.280	122.156
0.005	105.575	63.7792	62.6033	61.9869	61.4871	65.9075	62.5342	61.9621	61.6572	61.4273	61.3667
0.01	61.9953	35.3224	34.6189	34.2520	33.9532	36.6019	34.5802	34.2356	34.0570	33.9159	33.8831
0.02	34.3362	19.0147	18.6253	18.4228	18.2575	19.7247	18.6046	18.4133	18.3155	18.2365	18.2193
0.05	15.2175	8.44365	8.27481	8.18716	8.11551	8.75189	8.26604	8.18291	8.14079	8.10628	8.09908
0.10	8.34550	4.77567	4.68660	4.64037	4.60255	4.93828	4.68200	4.63810	4.61593	4.59767	4.59391
0.20	4.79874	2.91139	2.86370	2.83890	2.81868	2.99840	2.86125	2.83772	2.82585	2.81606	2.81405
0.50	2.61287	1.77874	1.75698	1.74567	1.73641	1.81839	1.75586	1.74511	1.73969	1.73521	1.73430
1.00	1.85375	1.39533	1.38301	1.37661	1.37137	1.41775	1.38238	1.37630	1.37323	1.37069	1.37017

Table 7: The RMIs from the simulated data fitted to AR(1) AR(2) and AR(3) models from Tables 4–6, respectively, on EWMA, MEWMA, and DMEWMA control charts

Chart	c_1	c_2	RMI		
			AR(1)	AR(2)	AR(3)
EWMA	0	-	0.6578	0.6564	0.6546
MEWMA	0.5	-	0.0340	0.0340	0.0342
MEWMA	1	-	0.0174	0.0175	0.0175
MEWMA	2	-	0.0087	0.0087	0.0088
MEWMA	10	-	0.0017	0.0016	0.0017
DMEWMA	0.5	0.5	0.0645	0.0643	0.0644
DMEWMA	1	1	0.0166	0.0165	0.0165
DMEWMA	2	1	0.0082	0.0083	0.0084
DMEWMA	2	2	0.0042	0.0041	0.0041
DMEWMA	10	2	0.0007	0.0006	0.0008
DMEWMA	10	10	0.0000	0.0000	0.0000

Table 8: The ARLs for the COVID-19 cases in Malaysia fitted to an AR(1) model on the various control charts with control limit [0, u]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
u	**0.0000587**	**785.7**	**1577.21**	**3158.66**	**15810.8**	**387.5**	**1660.25**	**3320.76**	**6562.8**	**32500.1**	**159657**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	362.611	336.706	302.092	274.019	245.107	360.946	299.878	272.484	256.030	241.061	237.960
0.002	355.472	308.941	255.357	217.655	183.434	351.845	252.058	215.691	195.817	178.928	175.521
0.005	334.965	247.465	174.433	134.779	104.831	326.878	170.532	132.898	115.074	101.221	98.5380
0.01	303.621	185.450	114.178	82.6779	61.4953	291.704	110.856	81.2814	68.5069	59.0708	57.2859
0.02	250.186	122.913	67.5746	46.8954	34.0394	238.522	65.2886	46.0217	38.2069	32.6154	31.5731
0.05	143.166	59.9580	30.4739	20.8047	15.0836	149.258	29.3712	20.4089	16.9158	14.4615	14.0076
0.10	60.6186	31.3965	16.0297	11.1490	8.27491	86.2112	15.4700	10.9500	9.19526	7.96221	7.73392
0.20	13.9560	15.3958	8.38948	6.12346	4.76243	41.5045	8.13148	6.03001	5.20126	4.61252	4.50281
0.50	1.43681	5.74709	3.75668	3.05058	2.59911	12.3764	3.67847	3.02036	2.74757	2.54765	2.50974
1.00	1.01381	2.92248	2.27708	2.02248	1.84831	4.72664	2.24975	2.01115	1.90679	1.82773	1.81246

Table 9: The ARLs for the COVID-19 cases in Japan fitted to an AR(2) model on various control charts with control limit [0, u]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
u	**0.000352**	**4708.7**	**9452**	**18929.4**	**94752**	**2322.5**	**9949.61**	**19900.8**	**39330**	**194769**	**956804**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	362.994	336.856	301.879	273.655	244.515	360.621	299.455	271.954	255.621	240.526	237.357
0.002	355.846	308.926	254.971	217.154	182.818	351.497	251.521	215.086	195.296	178.349	174.908
0.005	335.318	247.183	173.928	134.278	104.356	326.473	169.961	132.362	114.606	100.767	98.0804
0.01	303.941	185.037	113.733	82.3004	61.1796	291.243	110.389	80.8939	68.1755	58.7686	56.9873
0.02	250.449	122.508	67.2621	46.6571	33.8547	238.023	64.9742	45.7825	38.0061	32.4387	31.4003
0.05	143.317	59.7038	30.3209	20.6958	15.0028	148.826	29.2211	20.3011	16.8262	14.3842	13.9324
0.10	60.6816	31.2564	15.9513	11.0939	8.23414	85.9216	15.3937	10.8956	9.14992	7.92312	7.69597
0.20	13.9697	15.3309	8.35300	6.09731	4.74272	41.3606	8.09602	6.00423	5.17949	4.59357	4.48438
0.50	1.43728	5.72910	3.74547	3.04199	2.59226	12.3411	3.66753	3.01188	2.74015	2.54101	2.50324
1.00	1.01382	2.91700	2.27317	2.01925	1.84557	4.71788	2.24590	2.00794	1.90388	1.82505	1.80982

Table 10: The ARLs for the COVID-19 cases in Thailand fitted to an AR(3) model on various control charts with control limit [0, u]

δ	EWMA	MEWMA				DMEWMA					
	$c_1 = 0$	$c_1 = 0.5$	$c_1 = 1$	$c_1 = 2$	$c_1 = 10$	$c_1 = 0.5$, $c_2 = 0.5$	$c_1 = 1$, $c_2 = 1$	$c_1 = 2$, $c_2 = 1$	$c_1 = 2$, $c_2 = 2$	$c_1 = 10$, $c_2 = 2$	$c_1 = 10$, $c_2 = 10$
u	**0.0000456**	**609.43**	**1223.38**	**2450.04**	**12263.75**	**300.53**	**1287.78**	**2575.77**	**5090.47**	**25208.9**	**123839**
0.000	370	370	370	370	370	370	370	370	370	370	370
0.001	363.101	336.738	302.461	274.257	245.325	360.411	299.777	272.613	256.064	241.252	238.222
0.002	355.951	309.021	255.708	217.906	183.661	351.350	252.075	215.873	195.942	179.140	175.770
0.005	335.417	247.618	174.718	134.990	105.008	326.483	170.661	133.082	115.225	101.389	98.7148
0.01	304.031	185.631	114.385	82.8278	61.6129	291.434	110.995	81.4198	68.6221	59.183	57.3993
0.02	250.523	123.075	67.7055	46.9872	34.1083	238.402	65.3935	46.1090	38.2793	32.6812	31.6382
0.05	143.359	60.0557	30.5339	20.8459	15.1137	149.288	29.4247	20.4488	16.9488	14.4903	14.0357
0.10	60.6993	31.4495	16.0597	11.1697	8.29011	86.2682	15.4978	10.9702	9.21204	7.97678	7.74811
0.20	13.9736	15.4201	8.40329	6.13325	4.76977	41.5428	8.14452	6.03959	5.20935	4.61959	4.50969
0.50	1.43740	5.75379	3.76089	3.05378	2.60166	12.3872	3.68251	3.02351	2.75033	2.55013	2.51216
1.00	1.01383	2.92453	2.27855	2.02369	1.84933	4.72937	2.25117	2.01234	1.90786	1.82873	1.81344

Table 11: The RMIs from the COVID-19 cases in Malaysia, Japan and Thailand on EWMA, MEWMA and DMEWMA control charts

Chart	c_1	c_2	RMI		
			Malaysia	Japan	Thailand
EWMA	0	-	3.3331	3.3579	3.3309
MEWMA	0.5	-	2.1458	2.1483	2.1449
MEWMA	1	-	0.9600	0.9601	0.9603
MEWMA	2	-	0.5093	0.5088	0.5094
MEWMA	10	-	0.2127	0.2119	0.2128
DMEWMA	0.5	0.5	5.3782	5.3858	5.3697
DMEWMA	1	1	0.9112	0.9109	0.9107
DMEWMA	2	1	0.4897	0.4890	0.4897
DMEWMA	2	2	0.3110	0.3105	0.3109
DMEWMA	10	2	0.1786	0.1779	0.1787
DMEWMA	10	10	0.1535	0.1527	0.1536

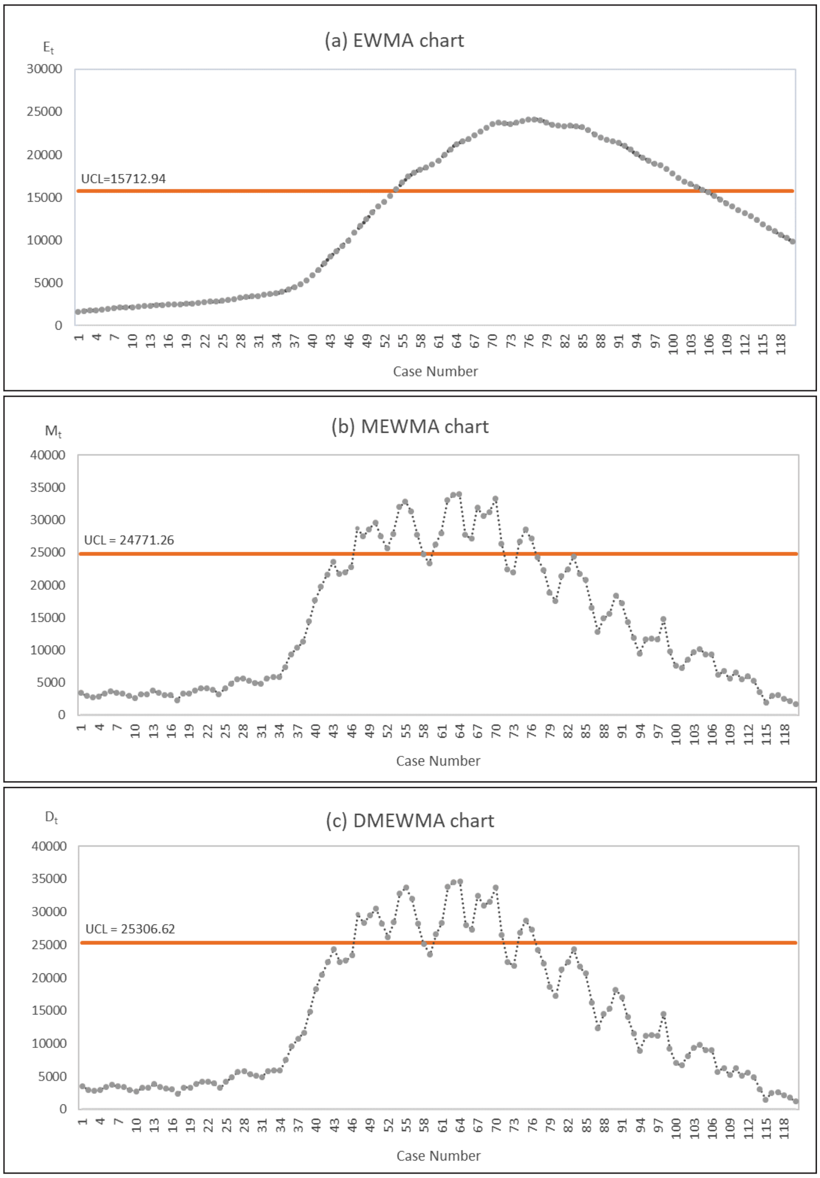

Figure 1: The COVID-19 Malaysia dataset fitted to an AR(1) process running on (a) an EWMA chart, (b) a MEWMA chart and (c) a DMEWMA chart.

In addition, the EWMA (Et), MEWMA (Mt) and DMEWMA (Dt) statistics with c_1, c_2 = 1, λ_1, λ_2 = 0.05 and control limits [0, UCL] for the Malaysian COVID-19 dataset fitted to an AR(1) model with μ_1 = 14082.65 and σ_1 = 10179.48 are presented in Figure 1. These results indicate that the MEWMA and DMEWMA charts can detect a shift at the 47th observation and on many occasions, while the EWMA scheme is found for the first time at the 54th observation. For Japan, these charts for the COVID-19 data fitted to an AR(2) model with μ_2 = 51189.33 and σ_2 = 25949.72 are illustrated in Figure 2. The results show that the MEWMA and DMEWMA charts could detect an abrupt shift at the 28th observation and many more, while the EWMA scheme is found for the first time at the 39th observation. In addition, the

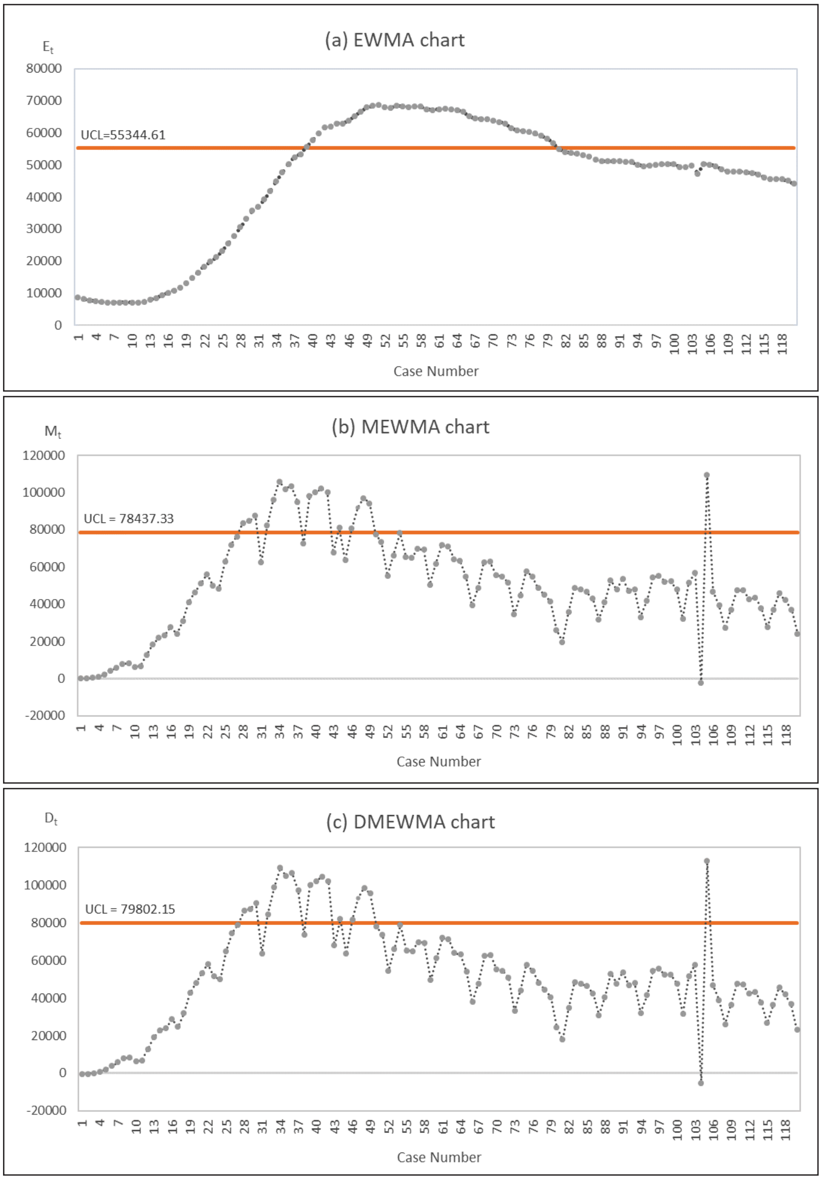

Figure 2: The COVID-19 Japan dataset fitted to an AR(2) process running on (a) an EWMA chart, (b) a MEWMA chart and (c) a DMEWMA chart.

MEWMA and DMEWMA charts can detect the 105th observation while the EWMA scheme cannot find it such that these results support the efficiency of the MEWMA and DMEWMA charts. Moreover, three charts for the COVID-19 cases in Thailand fitted to an AR(3) model with $\mu_3 = 16895.95$ and $\sigma_3 = 7423.41$ are plotted in Figure 3. For results, the MEWMA and DMEWMA charts can alert the first out-of-control at the 56th observation and many others, while the EWMA scheme is detected at the 70th observation. Hence, all results show that the MEWMA and DMEWMA charts are more effective than the EWMA chart. The DMEWMA chart can be detected similarly to the MEWMA chart such that this chart is determined $c_1 = c_2$ due to limitations of the UCL formula proposed by Alevizakos *et al.*, [11].

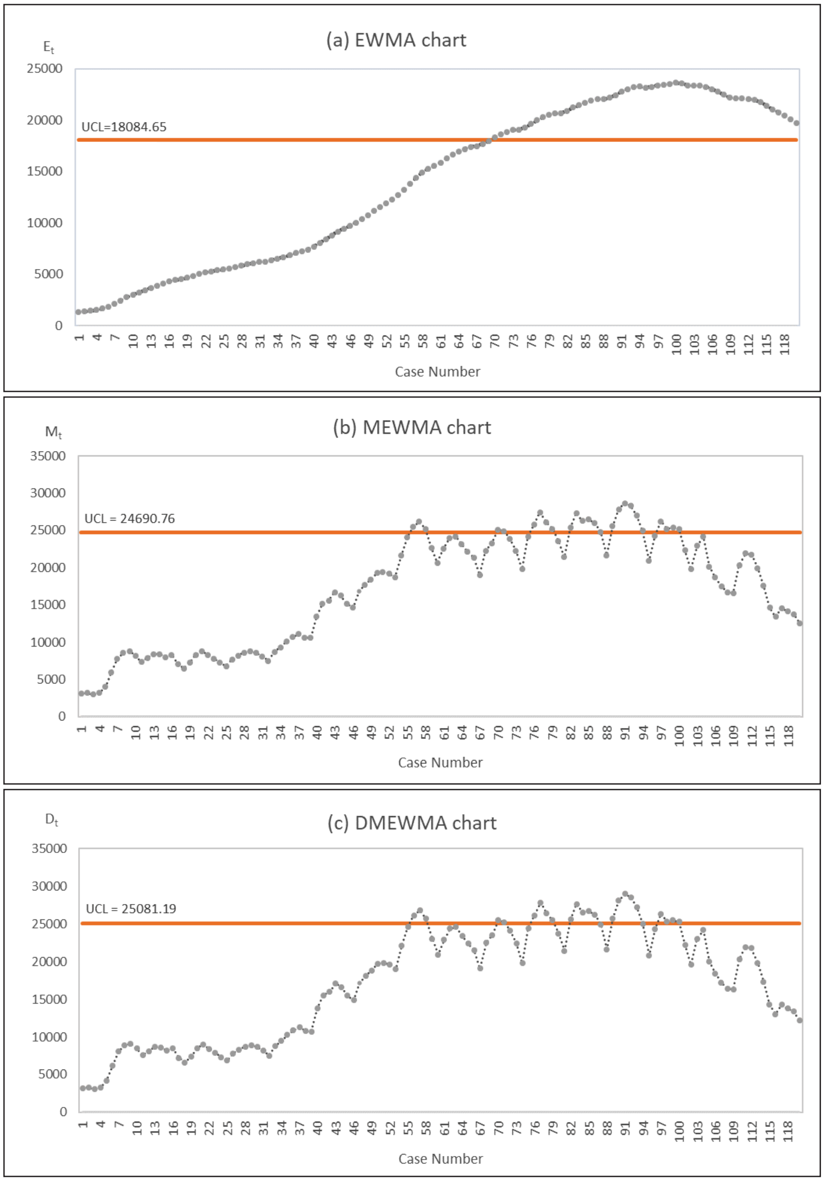

Figure 3: The COVID-19 Thailand dataset fitted to an AR(3) process running on (a) an EWMA chart, (b) a MEWMA chart and (c) a DMEWMA chart.

4 Conclusions

We establish an explicit formula for the ARL of an AR(p) model with exponential white noise on a DMEWMA control chart and apply it to analyze COVID-19 datasets from Malaysia, Japan and Thailand. The efficiency of the ARL using the explicit formula is validated against that of the ARL derived by using the well-known NIE method. The ARL of this DMEWMA control chart is used to compare performance with EWMA and MEWMA charts. The results indicate that the DMEWMA control chart with the same c_1 and c_2 performed better than the EWMA and MEWMA charts for small and moderate changes. Furthermore, the DMEWMA control chart becomes more effective for larger c_1 and c_2. In addition, this explicit formula is applied to COVID-19 datasets from Malaysia, Japan and Thailand such that these results are consistent with the simulated data. This explicit formula for the ARL can be applied to data on other pandemic diseases that may occur in the future, but it is limited to the case of exponential residuals in an AR model. Moreover, the limitation of the DMEWMA control chart is its decreased performance when large changes in the process mean are detected. For future research, the explicit formulas will be established for the ARL of the DMEWMA control chart with other models such as SAR, MA and ARIMA models for various real-life situations.

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Author Contributions

P.P.: investigation, methodology, data analysis, writing an original draft, reviewing and editing; S.S.: data curation, reviewing and editing; Y.A.: conceptualization, research design, reviewing and editing, funding acquisition, project administration. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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