Research Article

Explicit Formulas of ARL on Double Moving Average Control Chart for Monitoring Process Mean of ZIPINAR(1) Model with an Excessive Number of Zeros

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Abstract
Usually, the performance of control charts are widely measured by average run length (ARL). In this paper, the derivative explicit formulas of the ARL for double moving average (DMA) control chart are proposed for monitoring the process mean of zero-inflated Poisson integer-valued autoregressive first-order (ZIPINAR(1)) model. This model is fit when there are an excessive number of zeros in the count data. The performance of the DMA control chart is compared with the results of moving average and Shewhart control charts by considering from out of control average run length (ARL_1). The numerical results found that the DMA control chart performed better than other control charts in order to detect mean shift in the process. In addition, the real-world application of the DMA control chart for ZIPINAR(1) process is addressed.

Keywords: Average run length, Double moving average control chart, Zero-inflated Poisson

1 Introduction

The Poisson distribution has often been used for analyzing related data, particularly in quality control studies for industries in which the number of defects or nonconformities in a unit is commonly modelled using the Poisson distribution. However, this model does not provide a good fit to actual data when they contain a frequent or excessive number of zeros. For example, in a near zero-defect manufacturing environment, there are many zero-defect counts, even for fairly large sample sizes, and in such situations, the zero-inflated Poisson (ZIP) distribution is more appropriate and has been considered by several authors, see details on [1], [2].

In many cases, the Poisson distribution is not a good fit when there is a large number of zero counts, and thus alternative models have been proposed [3], [4]. Most researchers have mainly conducted studies on independent countable data but sometimes the data can be autocorrelated. In 1987, Osh and Alzaid [5] developed the Poisson integer-valued autoregressive (PINAR(1)) model which has often been used for count-related data by using a binomial thinning operator [6], [7]. However, this model does not provide a good fit to actual data when there is a frequent or excessive number of zero counts. Therefore, Jazi *et al.* [8] introduced ZIP with the first-order integer-valued autoregressive model (ZIPINAR(1)).

Statistical quality control is a statistical tool used in process control. Control charts are widely used in the quality control of the counting process for detecting changes in the process mean, thereby improving the quality of the process. It is a powerful tool with a clear purpose. A popular chart for detecting shifts in the process mean is the Shewhart control chart which uses only information in the last sample but ignores information available in the entire data sequence. Therefore, the Shewhart control chart performs poorly when detecting small process mean shifts [9].

In the past few decades, several control charts that can detect small process mean shifts have been proposed. For example, the cumulative sum (CUSUM) control chart based on the differences between sample values, and the average was introduced by Page [10].

Later, the exponentially weighted moving average (EWMA) control chart based on weighted previous information was proposed by Roberts [11]. Recently, Khoo [12] developed the moving average (MA) control chart based on the accumulative sum of information from the current and the previous sample. Meanwhile, Khoo and Wong [13] proposed the double moving average (DMA) control chart that can detect small changes and can be used with both continuous and discrete distributions. These are memory control charts that are superior to the Shewhart control chart for the detection of small to moderate shifts in the process mean. This is because they use information about the process available from the entire sequence of data points.

The performance of a control chart is usually evaluated via average run length (ARL), which is the expected number of points plotted on the control chart until an out of control signal is triggered. The average run length of a control chart when the process is in control stage denoted by ARL_0. The control chart is regarded as acceptable if ARL_0 is sufficiently large. Conversely, another performance measure in the out of control state is the average of the delay time or the out of control average run length (ARL_1), which is the expected number of observations from in control process until the control chart signals that the process is out of control. Ideally, the ARL_1 should be minimal.

There are several techniques to calculate the ARL. For example, a basic approach that is often used to test accuracy with other methods is Monte Carlo simulation which is simple for coding and testing the accuracy of the analytical approximation. However, it needed large number of observations, very time consuming, and difficult to obtain optimal values. The Markov chain approach is considered to be an effective alternative technique that approximates the performance measures using matrix inversion with Markov chain principles. Although there have not been any theoretical results recently on the convergence of this technique, it has been tested by direct comparison with Monte Carlo simulation results. The integral equation approach is the most advanced method currently available because it uses a mathematical integral formula.

In the literatures, several methods for evaluating ARL_0 and ARL_1 have been reported. Knoth [14] and Borror *et al.* [15] used the Markov chain approach for the ARL to examine the performance of the EWMA control chart for both skewed and heavy-tailed symmetric non-normal distributions. Sukparungsee [16] determined the performances of EWMA and CUSUM control charts with ARLs via the Markov chain approach for underlying ZIP processes. Crowder [17] used a numerical quadrature method to solve the exact integral equation for the ARL on an EWMA control chart for normally distributed data. Peerajit *et al.* [18] adopted the numerical integral equation method for the ARL on a CUSUM control chart. Suriyakat [19] applied the Fredholm integral equation for the ARL to detect the performance of EWMA control chart for an autoregressive model with exponential white noise. Areepong [20] studied explicit formulas for the ARL on an MA control chart for monitoring the number of defective products and showed that they were easy to use, calculate, and program. Areepong and Sukparungsee [21] analyzed closed-form formulas for the ARL on an MA control chart for a nonconforming ZIP and showed that it performs better as the value of the span (w) decreases. Chananet *et al.* [22] studied an approximate formula for the ARL on an MA control chart with zero-inflated negative binomial data and showed that the new formula is simple and easy to implement. Recently, Rakitzis *et al.* [23] studied ZIPINAR(1) processes via the CUSUM control chart which can track mean shift changes in manufacturing processes. They also discussed the influence of zero-inflated data on the control chart. Later, Areepong [24] studied explicit expressions for the ARL of an MA control chart for a ZIPINAR(1) model and showed that the performance of the MA control chart is better than the EWMA control chart. However, the numerical results for the ARL cannot usually be obtained analytically, and required intensively specialized software.

In this paper, explicit formulas are proposed for the ARL on DMA control chart to detect shifts in the mean process for ZIPINAR(1) model, and its efficiency of change detection is compared with MA and Shewhart control charts. The outline is organized as follows. Section 2 contains an overview of a ZIPINAR(1) model, the MA and DMA control charts for the ZIPINAR(1) model. Section 3 gives an overview of the derived explicit formulas of the ARL on a DMA control chart for the ZIPINAR(1) model. In Section 4, a comparison of the ARL performances of the DMA, MA, and Shewhart control charts is presented. Section 5 presents a real data application. Last, conclusions are provided in Section 6.

2 The ZIPINAR(1) Model

The first-order integer-valued autoregressive (INAR(1)) model is perfectly suited for modelling count data by making use of a thinning operator for the serially autocorrelated process. The thinning operator is generated by counting a series of Bernoulli distributed random variables. The INAR(1) model is defined as

$$N_t = \alpha \circ N_{t-1} + \varepsilon_t,$$

where N_t is the observable count at the time t, α is an INAR(1) parameter, ($\circ$) is the thinning operation at the time (t) performed independently, and ε_t is an innovation.

The ZIPINAR(1) model is the best fitting model for Poisson marginal distributions [24] and the innovation ε_t follows the ZIP distribution with mean $\frac{\lambda(1-\phi)}{1-\alpha}$, then $\varepsilon_t \sim Poi(\lambda,\phi)$ distribution has zero-inflated parameter (ϕ), where $\phi \in [0,1)$. This can be modeled as an INAR(1) process, for which the respective expectation and variance of the ZIPINAR(1) model can be written as

$$E[N_t] = \frac{\lambda(1-\phi)}{1-\alpha} \text{ and } V[N_t] = \frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{(1-\alpha^2)}.$$

Generally, the ZIPINAR(1) model can be changed by any unexpected occurrence, then the change-point model of this process can be described as follows. Assume that λ_0 and $\frac{\lambda_0(1-\phi)}{1-\alpha}$ are the in control parameter and λ_1 and $\frac{\lambda_1(1-\phi)}{1-\alpha}$ are the out of control parameter.

2.1 *The MA control chart for the ZIPINAR(1) model*

An MA control chart is based on the unweighted moving average and its flexible and easy to practical implement. Khoo [12] proposed an MA control chart for the number of nonconformities in a product inspection unit. Suppose individual observations, N_1, N_2,..., are collected from the ZIPINAR(1) model and follow an independent identically distributed (i.i.d.) sequence. The value of span w at the time i is defined as [12], then the MA_i statistic can be written as Equation (1)

$$MA_i = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} N_j & ; \ i < w \\ \frac{1}{w}\sum_{j=i-w+1}^{i} N_j & ; \ i \geq w. \end{cases} \quad (1)$$

When the process is in control, the respective mean and variance of MA_i are

$$E(MA_i) = \frac{\lambda(1-\phi)}{1-\alpha}$$

and

$$Var(MA_i) = \begin{cases} \frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i(1-\alpha^2)} & ; \ i \leq w \\ \frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w(1-\alpha^2)} & ; \ i > w \end{cases}.$$

The upper and lower control limits for the MA_i control chart are given by Equation (2) as

$$UCL/LCL = \begin{cases} \frac{\lambda(1-\phi)}{1-\alpha} \pm k\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i(1-\alpha^2)}} & ; \ i \leq w \\ \frac{\lambda(1-\phi)}{1-\alpha} \pm k\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w(1-\alpha^2)}} & ; \ i > w \end{cases} \quad (2)$$

where k is a coefficient of the control limit and is determined based on the desired ARL_0. The ZIPINAR(1) process on an MA control chart will signal an out of control event when $MA_i < LCL$ or $MA_i > UCL$

2.2 *The DMA control chart for the ZIPINAR(1) model*

The DMA control chart was proposed by Khoo and Wong [13]. The performance of DMA is better that the *MA* control chart when a magnitude of shifts is small. The observations of the DMA statistic are the collected double moving averages of the *MA* statistic. The DMA statistic is defined as Equation (3)

$$DMA_i = \begin{cases} \frac{MA_i + MA_{i-1} + MA_{i-2} + \ldots}{i} & ; i \leq w \\ \frac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ; w < i < 2w-1 \\ \frac{MA_i + MA_{i-1} + \ldots + MA_{i-w+1}}{w} & ; i \geq 2w-1, \end{cases} \quad (3)$$

where MA_i is the statistic of the MA control chart and w is the span at the time i. Then, it can be rewritten as Equation (4)

$$MA_i = \begin{cases} \dfrac{N_i + N_{i-1} + N_{i-2} + \ldots}{i} & ; i < w \\ \dfrac{N_i + N_{i-1} + \ldots + N_{i-w+1}}{w} & ; i \geq w. \end{cases} \quad (4)$$

Let observations $N_1, N_2 \ldots,$ be *i.i.d.* random variables in the ZIPINAR(1) model from Equation (4) which are collected moving averages of span w at the time i. The respective expectation and variance of the DMA control chart are

$$E(DMA_i) = \frac{\lambda(1-\phi)}{1-\alpha}$$

and

$$V(DMA_i) = \begin{cases} \dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)} \displaystyle\sum_{j=1}^{i} \frac{1}{j} & ; i \leq w \\ \dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)} \times \displaystyle\sum_{j=i-w+1}^{w-1} \frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right) & ; w < i < 2w-1 \\ \dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)} & ; i \geq 2w-1. \end{cases}$$

The upper and lower control limits of the DMA control chart for the ZIPINAR(1) processes are given by Equation (5)

$$UCL_i / LCL_i = \begin{cases} \dfrac{\lambda(1-\phi)}{1-\alpha} \pm H\sqrt{\dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)} \displaystyle\sum_{j=1}^{i} \frac{1}{j}} & ; i \leq w \\ \dfrac{\lambda(1-\phi)}{1-\alpha} \pm H\sqrt{\dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)} \displaystyle\sum_{j=i-w+1}^{w-1} \frac{1}{j} + (j-w+1)\left(\frac{1}{w}\right)} & ; w < i < 2w-1 \\ \dfrac{\lambda(1-\phi)}{1-\alpha} \pm H\sqrt{\dfrac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}} & ; i \geq 2w-1 \end{cases} \quad (5)$$

where H is a coefficient of the control limits of the DMA control chart for the ZIPINAR(1) process which is a constant that can be chosen.

3 *An Explicit Formula for ARL of DMA Control Chart for ZIPINAR(1) Model*

The performance for control charts is measured via the ARL comprising ARL_0 and ARL. The explicit formulas for evaluating these components can be analytically derived by the central limit theorem when given the out of control limit, see detail [12], [20]–[22], [24].

Let $ARL = n$ then the analytical explicit formulas can show as Equation (6)

$$\frac{1}{n} = \frac{1}{n} \text{ P(an out of control signal at the time } i \leq w) + \frac{1}{n} \text{ P(an out of control signal at the time } w < i < 2w-1) + \left[\frac{2-(2w-2)}{n}\right] \text{P(the out of control signal at the time } i \geq 2w-1). \quad (6)$$

According to Equation (6), the DMA statistics in terms of the out of control signals at the time i are replaced as Equation (7)

$$\frac{1}{n} = \frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(\frac{\sum_{j=1}^{i} MA_j}{i} > UCL_{i\leq w}\right) + P\left(\frac{\sum_{j=1}^{i} MA_j}{i} < LCL_{i\leq w}\right)\right]\right\}$$
$$+\frac{1}{n}\left\{\sum_{j=i-w+1}^{2w-2}\left[P\left(\frac{\sum_{j=i-w+1}^{i} MA_j}{w} > UCL_{w<i<2w-1}\right) + P\left(\frac{\sum_{j=i-w+1}^{i} MA_j}{w} < LCL_{w<i<2w-1}\right)\right]\right\}$$
$$+\left[\frac{n-(2w-2)}{n}\right]\left\{P\left(\frac{\sum_{j=i-w+1}^{i} MA_j}{w} > UCL_{i\geq 2w-1}\right) + P\left(\frac{\sum_{j=i-w+1}^{i} MA_j}{w} < LCL_{i\geq 2w-1}\right)\right\}. \quad (7)$$

By substituting the control limit of the DMA statistic from Equation (5) into Equation (7), then it can be rewritten as

$$\frac{1}{n} = \frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(\frac{\sum_{j=1}^{i} MA_j}{i} > \frac{\lambda(1-\phi)}{1-\alpha} + H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}\right)\right.\right.$$

$$+P\left(\frac{\sum_{j=1}^{i}MA_j}{i}<\frac{\lambda(1-\phi)}{1-\alpha}-H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}\right)\Bigg]\Bigg\}$$

$$+\frac{1}{n}\left(\sum_{j=i-w+1}^{2w-2}\left[P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w}>\frac{\lambda(1-\phi)}{1-\alpha}\right.\right.\right.$$

$$\left.+H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}\right)$$

$$+P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w}<\frac{\lambda(1-\phi)}{1-\alpha}\right.$$

$$\left.\left.\left.-H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}\right)\right]\right\}$$

$$+\left[\frac{n-(2w-2)}{n}\right]\left\{P\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w}>\frac{\lambda(1-\phi)}{1-\alpha}\right.\right.$$

$$\left.+H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}\right)$$

$$+\mathrm{P}\left(\frac{\sum_{j=i-w+1}^{i}MA_j}{w}<\frac{\lambda(1-\phi)}{1-\alpha}\right.$$

$$\left.\left.-H\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}\right)\right\}. \tag{8}$$

By applying the central limit theorem to derive the explicit formulas, Equation (8) can be rewritten as

$$\frac{1}{n}=\frac{1}{n}\left\{\sum_{i=1}^{w}\left[P\left(Z_1>\frac{UCL_{i\le w}+0.5-\frac{\mu_0}{1-\alpha_0}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.$$

$$\left.\left.+P\left(Z_1<\frac{LCL_{i\le w}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right]\right\}$$

$$+\frac{1}{n}\left\{\sum_{j=i-w+1}^{2w-2}\left[P\left(Z_2>\frac{UCL_{w<i<2w-1}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right.\right.$$

$$\left.\left.+P\left(Z_2<\frac{LCL_{w<i<2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right]\right\}$$

$$+\left[\frac{n-(2w-2)}{n}\right]\left\{P\left(Z_3>\frac{UCL_{i\ge 2w-1}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right)\right.$$

$$\left.+P\left(Z_3<\frac{LCL_{i\ge 2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right)\right\} \tag{9}$$

where

$$Z_1=\frac{\sum_{j=1}^{i}MA_j-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}},$$

$$Z_2=\frac{\sum_{j=i-w+1}^{i}MA_j-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}$$

and $Z_3=\frac{\sum_{j=1}^{i}MA_j-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}.$

According to Equation (9), let

$$A=\sum_{i=1}^{w}\left[P\left(Z_1>\frac{UCL_{i\le w}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.$$

$$\left.+P\left(Z_1<\frac{LCL_{i\le w}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right]$$

$$B=\sum_{j=i-w+1}^{2w-2}\left[P\left(Z_2>\frac{UCL_{w<i<2w-1}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right.$$
$$\left.+P\left(Z_2<\frac{LCL_{w<i<2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right]$$

and $C=P\left(Z_3>\frac{UCL_{i\geq 2w-1}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right)$

$$+P\left(Z_3<\frac{LCL_{i\geq 2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right). \tag{10}$$

Subsequently, the explicit formula for the ARL on a DMA control chart can be rewritten by substituting A, B, and C from Equation (10) into Equation (9). Then, it can be written as

$$\frac{1}{n}=\frac{1}{n}A+\frac{1}{n}B+\frac{n-(2w-2)}{n}C$$
$$nC=1-A-B+(2w-2)C.$$

Since $ARL=n$ the explicit formula of ARL can show as Equation (11)

$$ARL=((1-A)-B)C^{-1}+2(2w-2). \tag{11}$$

Consequently, the fully explicit formula for ARL can be shown as Equation (12)

$$ARL=\left[1-\sum_{i=1}^{w-1}\left[P\left(Z_1>\frac{UCL_{i\leq w}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)+P\left(Z_1<\frac{LCL_{i\leq w}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right]\right]$$

$$+P\left(Z_2<\frac{LCL_{w<i<2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\Bigg]$$

$$\times\left[P\left(Z_3>\frac{UCL_{i\geq 2w-1}+0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right)\right.$$

$$\left.+P\left(Z_3<\frac{LCL_{i\geq 2w-1}-0.5-\frac{\lambda(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda(1-\phi)(1+\alpha+\phi\lambda)}{w^2(1-\alpha^2)}}}\right)\right]^{-1}+(2w-2). \tag{12}$$

Proposition I. The in control process requires in control parameter $\lambda=\lambda_0$, thus the explicit formula for the ARL_0 on the DMA control chart is as following Equation (13)

$$ARL_0=\left[1-\sum_{i=1}^{w-1}\left[P\left(Z_1>\frac{UCL_{i\leq w}+0.5-\frac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right.\right.$$

$$\left.\left.+P\left(Z_1<\frac{LCL_{i\leq w}-0.5-\frac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\frac{1}{j}}}\right)\right]\right]$$

$$-\sum_{j=w+1}^{2w-2}\left[P\left(Z_2>\frac{UCL_{w<i<2w-1}+0.5-\frac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right.$$

$$\left.+P\left(Z_2<\frac{LCL_{w<i<2w-1}-0.5-\frac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\frac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(j-w+1)\left(\frac{1}{w}\right)}}\right)\right]$$

$$\times\left[P\left(Z_3 > \frac{UCL_{i\geq 2w-1} + 0.5 - \dfrac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{w^2(1-\alpha^2)}}}\right) + P\left(Z_3 < \frac{LCL_{i\geq 2w-1} - 0.5 - \dfrac{\lambda_0(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_0(1-\phi)(1+\alpha+\phi\lambda_0)}{w^2(1-\alpha^2)}}}\right)\right]^{-1} + (2w-2). \quad (13)$$

Proposition II. The process in the out of control state requires out 0f control parameter $\lambda = \lambda_1$ and $\lambda_1 = (1 + \delta)\lambda_0$, where δ is the magnitude of the shift, thus the explicit formula for the ARL_1 on the DMA control chart can be written as Equation (14)

$$ARL_1 = \left[1 - \sum_{i=1}^{w-1}\left[P\left(Z_1 > \frac{UCL_{i\leq w} + 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\dfrac{1}{j}}}\right) + P\left(Z_1 < \frac{LCL_{i\leq w} - 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{i^2(1-\alpha^2)}\sum_{j=1}^{i}\dfrac{1}{j}}}\right)\right]\right]$$

$$-\sum_{j=w+1}^{2w-2}\left[P\left(Z_2 > \frac{UCL_{w<i<2w-1} + 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\dfrac{1}{j} + (j-w+1)\left(\dfrac{1}{w}\right)}}\right) + P\left(Z_2 < \frac{LCL_{w<i<2w-1} - 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{w^2(1-\alpha^2)}\sum_{j=i-w+1}^{w-1}\dfrac{1}{j} + (j-w+1)\left(\dfrac{1}{w}\right)}}\right)\right]$$

$$\times\left[P\left(Z_3 > \frac{UCL_{i\geq 2w-1} + 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{w^2(1-\alpha^2)}}}\right) + P\left(Z_3 < \frac{LCL_{i\geq 2w-1} - 0.5 - \dfrac{\lambda_1(1-\phi)}{1-\alpha}}{\sqrt{\dfrac{\lambda_1(1-\phi)(1+\alpha+\phi\lambda_1)}{w^2(1-\alpha^2)}}}\right)\right]^{-1} + (2w-2). \quad (14)$$

4 Numerical Results

The explicit formulas for ARL_0 and ARL_1 on the DMA control chart for ZIPINAR(1) model were compared with the *MA* and Shewhart control charts. The numerical results for ARL_0 and ARL_1 on a DMA control chart were calculated via Equations (13) and (14), respectively, using the Mathematica® software [25]. For ARL_0 on the DMA control chart, the *MA* of the width (w) was varied from 2, 5, 10, 15, and 20. When ARL_0 was 370, the coefficient control limit (H) was equal to 3. The in-control parameter of the ZIPINAR(1) model, λ_0, was set as 1 and 5; α_0 as 0.1, 0.5, and 0.9, and ϕ as 0.5, 0.7, and 0.9. The out-of-control parameter of the ZIPINAR(1) model, λ_1, was set as $\lambda_0(1 + \delta)$, while the parameter magnitude value of δ was varied as 0.1, 0.3, 0.5, 0.7, 0.9, 1.0, 1.5, and 2.0. When $\lambda_0 = 1$, the DMA control chart showed better performance than the *MA* and Shewhart control charts for every level change because of minimal ARL_1 (Tables 1–3). We can see that for the DMA control chart, the optimal w value decreased when the change level increased. In contrast, when $\lambda_0 = 5$, the DMA control chart performed better than the *MA* and Shewhart control charts for all levels of changes (Tables 4–6). These results show that the performance of the charts depends on the data characteristics or the in-control parameters (λ_0, ϕ, α).

5 Real Application Data

Real world application of the Shewhart, MA, and DMA control charts in the monitoring of a ZIPINAR(1) model using the proposed method was conducted. The monthly animal submission numbers in animal health laboratories from a region in New Zealand and the number of sudden deaths in animal health laboratories during the period January 2003 to December 2009 [24] are studied.

The graphical displays of these control charts along with the data are provided in Figure 1. The results show that the Shewhart and MA control charts did not trigger an alarm, while the DMA control chart triggered the out-of-control signal at sample 72 for the first time. Consequently, the DMA control chart is more effective at process monitoring than the Shewhart and MA control charts under these conditions.

Table 1: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given $\lambda = 1$, $\phi = 0.5$ and $ARL_0 = 370$

α	δ	MA					DMA					Shewhart
		$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 1$
0.1	0.1	229.9	221.1	207.9	196.4	186.3	195.6	154.4	90.1	60.4	**51.7**	233.1
	0.3	100.3	82.2	63.7	52.9	46.5	64.0	29.5	**19.4**	25.1	32.9	108.2
	0.5	51.9	37.7	27.1	23.0	21.6	27.6	**12.3**	14.9	21.5	26.0	59.6
	0.7	31.0	21.1	15.6	14.6	15.1	14.9	**8.2**	13.0	17.0	17.9	37.3
	0.9	20.6	13.8	11.0	11.3	12.7	9.5	**6.7**	11.2	12.8	12.3	25.6
	1.0	17.5	11.6	9.7	10.4	11.9	7.9	**6.3**	10.3	11.1	10.5	21.8
	1.5	9.2	6.5	6.7	8.2	9.9	***4.3***	4.9	6.7	6.3	6.1	11.8
	2.0	6.1	4.8	5.6	7.1	8.6	***3.1***	4.1	4.7	4.5	4.5	7.9
0.5	0.1	227.2	214.8	197.1	182.3	169.8	196.3	141.8	73.3	49.2	**45.3**	231.7
	0.3	93.9	71.8	52.2	42.2	36.9	60.8	23.7	**17.7**	24.3	31.7	104.4
	0.5	46.1	30.7	21.3	18.4	18.1	25.0	**10.4**	14.3	19.6	21.8	55.6
	0.7	26.5	16.7	12.4	12.3	13.5	13.2	**7.4**	12.0	13.9	13.5	33.8
	0.9	17.2	10.8	9.1	10.0	11.7	8.3	**6.2**	9.8	9.7	9.1	22.7
	1.0	14.3	9.1	8.2	9.4	11.1	6.9	**5.9**	8.6	8.3	7.8	19.2
	1.5	7.4	5.3	6.0	7.5	9.0	***3.8***	4.6	5.2	4.9	4.9	10.1
	2.0	4.9	4.0	5.1	6.4	7.4	***2.8***	3.7	3.7	3.6	3.7	6.6
0.9	0.1	202.1	165.3	126.3	102.5	86.9	161.0	66.2	**27.4**	28.1	35.4	218.0
	0.3	55.1	29.7	18.3	16.0	16.3	29.2	**9.4**	13.7	16.5	16.2	75.9
	0.5	20.6	10.5	8.7	9.9	11.7	9.8	**6.3**	8.3	7.6	7.3	32.5
	0.7	10.3	6.0	6.5	8.0	9.4	5.1	**5.0**	5.2	5.0	5.0	17.1
	0.9	6.3	4.4	5.4	6.6	7.4	**3.5**	3.9	3.8	3.8	3.8	10.4
	1.0	5.2	3.9	5.0	6.0	6.5	**3.1**	3.5	3.4	3.4	3.4	8.5
	1.5	2.8	2.8	3.5	3.7	3.7	**2.1**	2.3	2.3	2.3	2.3	4.1
	2.0	2.1	2.3	2.5	2.6	2.6	**1.7**	1.7	1.8	1.8	1.8	2.7

Table 2: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given $\lambda = 1$, $\phi = 0.7$ and $ARL_0 = 370$

α	δ	MA					DMA					Shewhart
		$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 1$
0.1	0.1	232.9	228.0	220.4	213.5	207.1	194.1	169.9	119.6	85.1	**68.6**	234.6
	0.3	107.9	96.2	81.9	71.9	64.9	67.5	40.2	**24.2**	26.8	34.1	112.5
	0.5	59.4	48.7	38.4	32.9	30.1	30.9	16.5	**16.0**	23.0	30.3	64.2
	0.7	37.2	28.9	22.4	19.9	19.3	17.4	***10.1***	14.0	20.4	25.0	41.4
	0.9	25.6	19.4	15.3	14.5	15.0	11.3	***7.7***	12.7	17.5	19.5	29.1
	1.0	21.9	16.4	13.2	12.9	13.8	9.5	***7.0***	12.1	16.0	17.2	25.1
	1.5	12.0	9.1	8.3	9.3	10.9	***5.1***	5.4	9.3	10.2	9.8	14.1
	2.0	8.1	6.4	6.6	7.9	9.6	***3.6***	4.6	7.1	7.0	6.7	9.6
0.5	0.1	231.3	224.2	213.5	203.8	195.3	195.3	148.0	81.4	54.4	**48.2**	233.8
	0.3	103.6	88.0	70.8	60.1	53.2	62.1	26.4	**18.5**	24.7	32.4	110.1
	0.5	55.1	42.0	31.2	26.4	24.4	26.1	**11.3**	14.6	20.6	24.1	61.6
	0.7	33.5	24.0	17.9	16.3	16.4	14.0	**7.8**	12.6	15.5	15.6	39.0
	0.9	22.6	15.8	12.4	12.3	13.4	8.9	**6.5**	10.5	11.2	10.6	27.0
	1.0	19.1	13.3	10.9	11.2	12.5	7.4	**6.1**	9.5	9.6	9.0	23.1
	1.5	10.2	7.4	7.3	8.5	10.2	***4.1***	4.8	6.0	5.6	5.4	12.7
	2.0	6.8	5.3	5.9	7.4	9.0	***3.0***	3.9	4.2	4.1	4.0	8.5
0.9	0.1	216.2	191.7	161.1	139.1	122.9	177.7	96.4	40.5	**32.9**	37.5	225.9
	0.3	73.7	46.5	29.7	23.9	22.1	41.4	**13.1**	15.3	21.3	24.4	90.9
	0.5	31.4	17.4	12.3	12.2	13.5	14.9	**7.4**	11.6	12.3	11.4	43.7
	0.7	16.6	9.3	8.2	9.5	11.3	7.7	**6.0**	7.9	7.4	7.1	24.7
	0.9	10.3	6.3	6.6	8.2	9.8	**5.0**	**5.0**	5.6	5.3	5.3	15.8
	1.0	8.5	5.5	6.2	7.7	9.2	**4.2**	4.7	4.9	4.7	4.7	13.1
	1.5	4.4	3.7	4.8	5.7	6.3	**2.6**	3.2	3.1	3.1	3.1	6.5
	2.0	3.0	3.0	3.9	4.3	4.4	**2.1**	2.4	2.4	2.4	2.4	4.3

Table 3: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given λ = 1, ϕ = 0.9 and ARL_0 = 370

α	δ	MA					DMA					Shewhart
		w = 2	**w = 5**	**w = 10**	**w = 15**	**w = 20**	**w = 2**	**w = 5**	**w = 10**	**w = 15**	**w = 20**	**w = 1**
0.1	0.1	235.3	233.8	231.4	229.1	226.8	192.9	185.1	162.6	137.7	***117.1***	235.8
	0.3	114.5	110.4	104.3	99.1	94.7	70.7	58.1	40.0	**34.9**	37.8	116.0
	0.5	66.5	62.1	56.3	52.1	48.9	34.3	25.7	**20.4**	24.6	32.1	68.1
	0.7	43.5	39.7	35.1	32.3	30.6	20.1	**14.9**	15.4	22.0	30.0	45.1
	0.9	31.0	27.8	24.4	22.6	21.9	13.4	**10.4**	13.5	20.6	27.9	32.4
	1.0	26.8	23.9	21.0	19.7	19.3	11.3	**9.1**	12.9	20.0	26.7	28.1
	1.5	15.4	13.5	12.2	12.2	12.9	6.2	**5.9**	11.0	16.8	20.7	16.3
	2.0	10.6	9.4	8.9	9.4	10.5	***4.3***	4.8	9.7	13.9	15.8	11.2
0.5	0.1	234.8	232.5	228.9	225.5	222.2	198.2	186.2	154.4	123.3	**101.2**	235.6
	0.3	113.0	106.9	98.4	91.5	85.9	74.0	55.45	34.9	**31.7**	36.3	115.2
	0.5	64.8	58.5	50.9	45.8	42.4	35.9	23.8	**18.9**	24.2	32.1	67.2
	0.7	41.9	36.6	30.9	27.9	26.3	20.9	13.8	**15.1**	22.1	29.6	44.1
	0.9	29.6	25.2	21.2	19.5	19.1	13.8	**9.8**	13.5	20.5	26.6	31.6
	1.0	25.5	21.6	18.2	17.0	17.0	11.6	**8.6**	13.0	19.7	24.9	27.3
	1.5	14.4	12.0	10.7	11.0	12.0	6.3	**5.9**	11.0	15.4	17.2	15.7
	2.0	9.8	8.3	8.0	8.8	10.1	***4.3***	4.9	9.5	11.9	12.1	10.7
0.9	0.1	229.8	220.7	207.2	195.4	185.2	194.6	152.8	88.5	59.3	**51.1**	233.0
	0.3	99.9	81.5	62.9	52.2	45.8	63.1	28.8	**19.2**	25.0	32.8	108.0
	0.5	51.6	37.3	26.7	22.7	21.4	27.1	12.1	14.8	21.3	25.7	59.4
	0.7	30.8	20.9	15.4	14.4	15.0	14.6	**8.1**	13.0	16.8	17.6	37.1
	0.9	20.5	13.6	10.9	11.2	12.6	9.3	**6.6**	11.1	12.5	12.1	25.4
	1.0	17.2	11.4	9.6	10.3	11.9	7.8	**6.2**	10.2	10.9	10.3	21.7
	1.5	9.1	6.5	6.7	8.1	9.8	**4.3**	4.9	6.6	6.2	6.0	11.7
	2.0	6.1	4.8	5.6	7.1	8.5	**3.1**	4.1	4.7	4.5	4.4	7.8

Table 4: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given λ = 5, ϕ = 0.5 and ARL_0 = 370

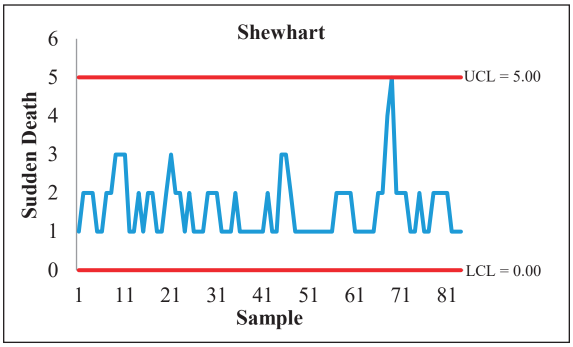
α	δ	MA					DMA					Shewhart
		w = 2	**w = 5**	**w = 10**	**w = 15**	**w = 20**	**w = 2**	**w = 5**	**w = 10**	**w = 15**	**w = 20**	**w = 1**
0.1	0.1	335.3	333.9	331.6	329.3	327.1	312.9	304.0	276.6	241.9	**208.1**	335.8
	0.3	273.7	264.6	250.9	238.6	227.7	222.3	181.2	110.6	73.0	**59.1**	276.9
	0.5	223.1	205.8	182.2	163.8	149.1	158.5	103.4	49.9	**37.5**	39.5	229.5
	0.7	182.3	159.2	131.4	112.3	98.7	114.4	61.4	29.6	**28.9**	35.5	191.5
	0.9	149.8	123.7	95.9	79.0	68.0	84.1	39.1	**21.9**	26.1	33.9	161.0
	1.0	136.1	109.4	82.56	67.1	57.6	72.7	32.0	**19.9**	25.3	33.2	148.0
	1.5	87.0	62.3	42.9	34.3	30.2	37.9	**15.3**	15.8	22.6	28.4	100.1
	2.0	58.8	38.8	26.0	21.7	20.5	22.3	**10.1**	14.1	19.6	22.0	71.0
0.5	0.1	334.7	332.4	328.7	325.1	321.6	313.9	299.6	258.7	212.4	**173.2**	335.5
	0.3	269.8	255.7	235.2	218.0	203.4	220.3	161.3	83.8	54.4	**47.9**	274.9
	0.5	215.4	190.0	158.6	136.4	120.1	153.0	83.0	36.4	**31.4**	36.8	225.4
	0.7	171.7	139.9	106.6	86.6	73.8	107.0	46.1	**23.2**	26.6	34.4	185.5
	0.9	137.4	103.7	73.5	57.8	49.0	76.2	28.4	**18.6**	24.9	32.5	153.6
	1.0	123.1	89.8	61.8	48.3	41.1	64.9	23.2	**17.5**	24.2	31.3	140.2
	1.5	74.1	46.9	30.0	24.1	22.2	31.8	**11.6**	14.8	20.5	23.3	91.2
	2.0	47.6	27.7	18.1	16.0	16.3	18.1	**8.3**	13.0	15.8	15.7	62.4
0.9	0.1	329.3	319.3	304.0	290.3	277.9	305.6	251.7	154.2	97.7	**73.1**	332.7
	0.3	238.0	193.2	146.2	117.6	98.8	178.3	71.6	**28.3**	28.4	35.6	257.5
	0.5	162.1	107.7	67.9	50.4	41.6	97.9	26.1	**17.7**	24.5	31.1	192.9
	0.7	109.4	61.9	35.8	27.1	24.0	55.6	**13.8**	15.5	21.2	23.6	143.1
	0.9	75.0	38.0	21.8	17.9	17.7	33.6	**9.7**	13.9	16.7	16.4	106.6
	1.0	62.8	30.5	17.9	15.5	16.0	26.7	**8.6**	13.0	14.5	13.6	92.4
	1.5	28.7	13.0	9.6	10.6	12.3	10.8	**6.4**	8.3	7.7	7.4	48.1
	2.0	15.6	7.6	7.3	8.8	10.4	6.0	**5.3**	5.5	5.3	5.3	27.9

Table 5: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given $\lambda = 5$, $\phi = 0.7$ and $ARL_0 = 370$

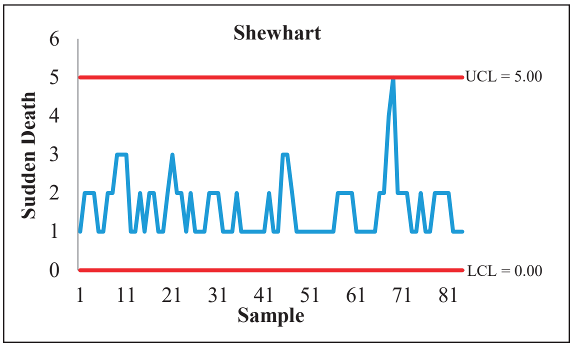
α	δ	MA					DMA					Shewhart
		$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 1$
0.1	0.1	335.8	335.2	334.0	333.0	331.9	311.9	307.7	293.9	274.3	**252.1**	336.0
	0.3	277.1	272.7	265.7	259.2	253.1	223.7	202.1	151.7	111.1	**87.4**	278.6
	0.5	230.0	221.1	207.9	196.4	186.3	163.2	130.0	78.2	54.6	**48.7**	233.1
	0.7	192.2	179.6	162.1	148.0	136.5	121.4	85.2	45.9	**36.4**	38.9	196.9
	0.9	161.9	146.7	127.1	112.5	101.5	92.1	57.9	31.3	**29.7**	35.7	167.7
	1.0	149.0	132.9	113.0	98.6	88.2	80.8	48.4	**27.1**	28.0	34.8	155.3
	1.5	101.4	84.0	65.9	55.1	48.5	45.1	23.5	**18.1**	24.3	32.1	108.9
	2.0	72.3	56.4	42.1	35.0	31.4	27.9	**14.4**	15.4	22.5	29.0	79.80
0.5	0.1	335.5	334.4	332.6	330.8	329.1	313.2	306.2	284.2	255.0	**224.9**	335.9
	0.3	275.1	268..0	256.9	246.8	237.7	224.1	190.4	125.3	85.0	**67.2**	277.6
	0.5	225.9	211.9	192.2	176.1	162.8	161.8	114.0	58.8	42.2	**41.8**	231.0
	0.7	186.3	167.2	142.8	125.0	111.7	118.5	70.2	34.3	**30.8**	36.44	193.7
	0.9	154.6	132.4	107.0	90.3	78.9	88.4	45.6	**24.5**	27.0	34.5	163.7
	1.0	141.2	118.2	93.1	77.5	67.3	76.8	37.6	**21.8**	26.0	33.8	151.0
	1.5	92.5	69.9	50.3	40.5	35.5	41.2	17.9	**16.4**	23.3	30.2	103.6
	2.0	63.8	44.6	30.9	25.4	23.4	24.7	**11.4**	14.6	20.9	24.9	74.4
0.9	0.1	332.8	327.8	319.8	312.3	305.2	309.7	280.3	209.9	150.6	**113.4**	334.5
	0.3	258.1	230.7	195.7	170.1	150.7	201.8	113.5	46.9	**35.3**	38.5	268.7
	0.5	194.1	152.0	110.9	87.7	73.3	126.8	47.7	**22.7**	26.5	34.4	213.2
	0.7	144.6	99.9	65.3	49.5	41.4	80.6	24.5	**17.6**	24.4	31.2	168.8
	0.9	108.3	67.4	41.3	31.5	27.5	52.9	**15.3**	15.8	22.3	26.3	134.1
	1.0	94.2	56.0	33.8	26.2	23.6	43.5	**12.9**	15.2	20.9	23.6	119.9
	1.5	49.8	25.7	16.0	14.5	15.3	19.0	**7.9**	12.4	13.6	12.8	71.0
	2.0	29.3	14.5	10.5	11.1	12.8	10.4	**6.5**	9.2	8.7	8.2	45.0

Table 6: Comparison of ARL of the Shewhart, MA and DMA charts for ZIPINAR(1) model given $\lambda = 5$, $\phi = 0.9$ and $ARL_0 = 370$

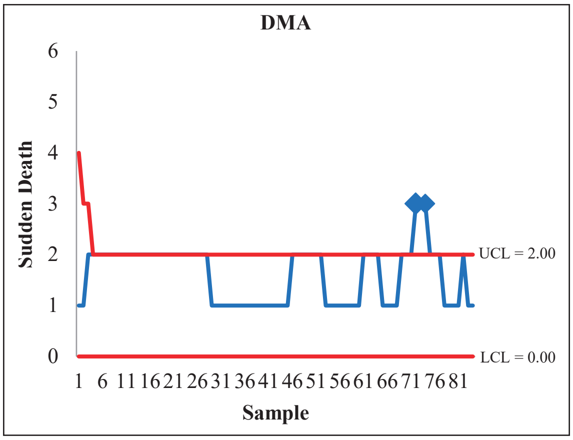
α	δ	MA					DMA					Shewhart
		$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 2$	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 1$
0.1	0.1	336.2	335.9	335.6	335.3	335.0	311.2	310.1	306.1	300.0	**292.2**	336.2
	0.3	279.3	278.0	276.0	274.0	272.2	224.6	218.2	198.8	174.9	**152.4**	279.6
	0.5	234.6	232.0	227.8	224.0	220.3	166.4	155.4	127.4	101.5	**84.6**	235.4
	0.7	199.0	195.2	189.2	183.7	178.7	126.4	112.8	84.1	64.7	**56.5**	200.3
	0.9	170.5	165.6	158.3	151.7	145.9	98.1	83.7	58.4	46.2	**44.4**	172.1
	1.0	158.3	153.0	145.2	138.3	132.3	87.1	72.7	49.8	**40.6**	41.1	160.1
	1.5	112.7	106.2	97.2	90.1	84.3	51.4	39.3	**27.4**	28.0	34.1	115.0
	2.0	83.8	77.2	68.6	62.3	57.7	33.3	24.3	**19.5**	24.2	31.9	86.3
0.5	0.1	336.0	335.7	335.2	334.7	334.2	312.6	310.7	304.1	294.1	281.9	336.1
	0.3	278.7	276.6	273.3	270.1	267.0	226.8	216.1	186.1	153.4	**127.2**	279.4
	0.5	233.3	229.0	222.4	216.2	210.4	168.3	150.4	111.0	82.3	**67.9**	234.8
	0.7	197.2	190.9	181.4	173.0	165.6	127.7	106.2	69.6	51.7	**47.5**	199.4
	0.9	168.1	160.3	148.9	139.3	131.1	98.9	76.8	47.2	**38.1**	39.7	171.0
	1.0	155.8	147.3	135.2	125.3	117.2	87.6	65.9	40.1	**34.2**	37.7	158.9
	1.5	109.5	99.4	86.6	77.3	70.4	51.2	34.1	**23.0**	26.1	33.5	113.4
	2.0	80.4	70.5	58.9	51.5	46.6	32.8	20.7	**17.6**	23.7	31.6	84.5
0.9	0.1	335.3	333.8	331.4	329.2	326.9	312.6	303.6	276.0	241.0	**207.0**	335.7
	0.3	273.6	264.4	250.5	238.2	227.2	221.6	180.3	109.6	72.3	**58.7**	276.8
	0.5	222.9	205.4	181.7	163.2	148.4	157.7	102.6	49.4	**37.2**	39.3	229.4
	0.7	182.1	158.8	130.9	111.7	98.0	113.7	60.7	29.3	**28.8**	35.4	191.4
	0.9	149.5	123.3	95.4	78.4	67.5	83.5	38.6	**21.7**	26.0	33.8	160.8
	1.0	135.8	109.0	82.1	66.6	57.1	72.1	31.6	**19.8**	25.2	33.1	147.8
	1.5	86.8	61.9	42.6	34.0	30.0	37.5	**15.1**	15.7	22.6	28.2	99.9
	2.0	58.5	38.5	25.8	21.5	20.3	22.1	**10.0**	14.1	19.4	21.8	70.8



(a) Shewhart control chart



(b) MA control chart



(c) DMA control chart

Figure 1: Performance comparisons of control charts for the monthly numbers of sudden death.

6 Conclusions

The explicit formulas for ARL_0 and ARL_1 on the DMA control chart were derived and their accuracy determined by comparison with the results obtained by Monte Carlo simulation. The tables of ARL were constructed for various shifts in the process mean underlying a ZIPINAR(1) model. A comparison of the DMA control chart with MA and Shewhart control charts were made in terms of ARL, in which the performance of the DMA control chart is better than other benchmark control chart in the process mean. The explicit formulas for ARL_0 and ARL_1 on the DMA control chart are easy to calculate and implement for real-world applications.

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