Research Article

Statistical Inference on the Ratio of Delta-Lognormal Coefficients of Variation

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Abstract
The coefficient of variation is useful to measure and compare the dispersion of the data when different units are used in different datasets. This article aims to propose new confidence intervals for the ratio of two independent coefficients of variation with delta-lognormal distribution. The proposed methods include the concept of the generalized confidence interval and the method of variance estimate recovery. They are applied with three methods, variance stabilizing transformation, Wilson score method, and Jeffreys method. The performance of the confidence intervals was assessed by the coverage probabilities and the expected lengths via the Monte Carlo simulation. The outcomes of the simulation study showed that the generalized confidence interval is appropriate to construct the confidence interval for the ratio of delta-lognormal coefficients of variation. Two rainfall datasets from Nakhon Ratchasima, Thailand are used to demonstrate the proposed confidence intervals.

Keywords: Generalized confidence interval, Method of variance estimates recovery, Variance stabilizing transformation, Wilson score method, Jeffreys method, Rainfall

1 Introduction

Nowadays, the global climate is changing with the causes being El Niño, the warm phase of the El Niño Southern Oscillation, together with the Madden-Julian Oscillation (MJO) in the Indian and Pacific oceans and the Indian Ocean Dipole (IOD) in the Indian Ocean. The MJO and the IOD influence the seasonal and annual rainfall, respectively [1]. Especially, Thailand is located in a tropical area near the equator and is directly affected by these phenomena. Moreover, it is influenced by the Southwest Monsoon current during the rainy season and the Northeast Monsoon current during the cold season. Furthermore, Thailand is located between the source of tropical cyclones in both the east (the Pacific Ocean and the South China Sea) and the west (the Bay of Bengal and the Andaman Sea). These storms move through Thailand around three to four times a year, mainly through the north and northeast of the country. On many occasions, there has been heavy rain that has often caused flooding and resulted in loss of life and damage to property. Therefore, this study on the variability of rainfall amount in each area by measuring the coefficient of variation is particularly important and could be useful to predict future rainfall and thereby prevent flooding in areas particularly affected by heavy rain. In addition, the ratio of the coefficients of variation of rainfall between two areas is of interest. Furthermore, the applications of rainfall data can be even more interesting, such as Ananthakrishnan and Soman [2] illustrated the relationship between the accumulated percentage of the rain amount and the number of rain days in a rainfall series using a normalized rainfall curve (NRC) and Shimizu [3] proposed a probability model to represent rainfall data as such data usually includes zero observations and a

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few datasets with delta-lognormal distribution. For delta-lognormal distribution, it is a combination of lognormal distribution and zero observations which is a binomial proportion. This distribution is often applied to many researches; see, e.g., [4]–[10].

The coefficient of variation is the statistical measure of the dispersion of data when the different data series have different units or awfully different mean. It is defined as the ratio of the standard deviation to the mean. The coefficient of variation is widely used in statistical inference, such as in the construction of confidence intervals which are investigated under normal and non-normal distribution. For normal distribution, there are several researchers who have studied methods to construct the confidence intervals for parameters of interest. This can be seen in the researches of Wong and Wu [11], Tian [12], Donner and Zou [13], Wongkhao *et al*. [14], and Hayter [15]. Moreover, non-normal distributions must be considered. For instance, Sangnawakij and Niwitpong [16] constructed confidence intervals for coefficients of variation in the two parameter exponential distributions, Niwitpong [17] suggested new confidence interval for the coefficient of variation of a lognormal distribution with restricted parameter, and Yosboonruang *et al*. [18], [19] presented the methods to construct confidence intervals for the coefficient of variation with a delta-lognormal distribution. In addition, the ratio of the coefficients of variation must be regarded when constructing confidence intervals, for example, Verrill and Johnson [20] obtained confidence interval for the ratio of coefficients of variation in a normal distribution, Buntao and Niwitpong [21] used two concepts, the generalized pivotal approach (GPA) and the method of variance estimate recovery (MOVER) based on Wald interval, to construct confidence intervals for the ratio of coefficients of variation of a delta-lognormal distribution, Nam and Kwon [22] proposed confidence intervals of the ratio of two coefficients of variation for lognormal distributions including Wald-Type method, Fieller-Type method, log method, and MOVER, and Hasan and Krishnamoorthy [23] proposed confidence intervals for the ratio of coefficients of variation of two lognormal distributions based on MOVER and fiducial approach. It can be seen that several researches used inference statistics for the coefficient of variation of two populations in terms of the ratio of the coefficients of variation.

This article is interested in inference for the ratio of two independent coefficients of variation of two delta-lognormal distributions. From the concept of Verrill and Johnson [20] that applied rainfall series with the confidence intervals for ratio of coefficients of variation, this article emphasized ratio of coefficients of variation to compare the dispersion of rainfall in two flooding areas by establishing confidence intervals using new methods which are based on the concept of the generalized confidence interval (GCI) and MOVER based on variance stabilizing transformation (VST), Wilson score method, and Jeffreys method. The next section presents two methods to construct confidence interval. Then, a simulation study and an empirical study are used to illustrate the performance of confidence intervals. Finally, the conclusion is presented in section 4.

2 Method

Given $\boldsymbol{X}_{ij} = (X_{i1}, X_{i2}, \ldots, X_{in_i}),\ i = 1,2,\ j = 1,2,...,n_i$ be a vector of random sample that contains zero and positive observed values. The zero observed values ($n_{i(0)}$) have a binomial distribution and the skewed positive observed values ($n_{i(1)}$) have a lognormal distribution of which $n_i = n_{i(0)} + n_{i(1)}$. Aitchison [24] described the distribution of such observations is a delta-lognormal distribution with $X_{ij} : \Delta(\delta_i, \mu_i, \sigma_i^2)$. The probability density function of delta-lognormal distribution is expressed by de la Mare [25] as

$$f\left(x_{ij};\delta_i,\mu_i,\sigma_i^2\right) = \left(1-\delta_i\right)\mathrm{I}_0\left[x_{ij}\right] + \delta_i \frac{1}{x_{ij}\sqrt{2\pi}\sigma_i} \times \exp\left\{-\frac{1}{2}\left[\frac{\ln\left(x_{ij}\right)-\mu_i}{\sigma_i}\right]^2\right\}\mathrm{I}_{(0,\infty)}\left[x_{ij}\right], \tag{1}$$

where δ_i is a probability of positive values $\left(P(X_{ij} > 0)\right)$, μ_i and σ_i^2 are mean and variance of positive observations distribution which is a lognormal distribution, $\mathrm{I}_0[x_{ij}]$ is an indicator function such as the value is 1 when $x_{ij} = 0$ and 0 otherwise, and $\mathrm{I}_{(0,\infty)}[x_{ij}]$ has the value 0 when $x_{ij} = 0$ and 1 when $x_{ij} > 0$. By Equation (1), the first term is a probability mass function of a binomial distribution and the second term is a probability density function of lognormal distribution. Let $Y_{ij} = \ln(x_{ij})$ has a normal distribution with $Y_{ij} : \mathrm{N}(\mu_i, \sigma_i^2)$. Aitchison [24] derived mean and variance of a delta-lognormal distribution as [Equations (2) and (3)]

$$E(X_{ij})\delta_i \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right) \tag{2}$$

and

$$Var\left(X_{ij}\right) = \delta_i \exp\left(2\mu_i + \sigma_i^2\right)\left[\exp\left(\sigma_i^2\right) - \delta_i\right] \tag{3}$$

Then, the coefficient of variation which is a ratio of standard deviation and mean of X_{ij}, denoted by η_i, can be expressed [Equation (4)]

$$\eta_i = \frac{\sqrt{Var\left(X_{ij}\right)}}{E\left(X_{ij}\right)} = \sqrt{\frac{\exp\left(\sigma_i^2\right) - \delta_i}{\delta_i}}. \tag{4}$$

Since the ratio of coefficients of variation was of interest and X_{ij} are independent, the ratio of coefficients of variation is simply [Equation (5)]

$$\zeta = \frac{\eta_1}{\eta_2} = \sqrt{\frac{\exp\left(\sigma_1^2\right) - \delta_1}{\delta_1}} \Bigg/ \sqrt{\frac{\exp\left(\sigma_2^2\right) - \delta_2}{\delta_2}}. \tag{5}$$

To construct the confidence intervals for the ratio of two independent coefficients of variation of the delta-lognormal distribution, two methods comprised of GCI and MOVER based on VST, Wilson score method, and Jeffreys method are investigated next.

2.1 *The generalized confidence interval*

GCI is used to construct the confidence intervals using the generalized pivotal quantity (GPQ). This method was recommended by Tsui and Weerahandi [26]. Given $\boldsymbol{X}_{ij} = (X_{i1}, X_{i2}, \ldots, X_{in_i})$ is a vector of random samples with the probability density functions $f_{\boldsymbol{X}_{ij}}(\boldsymbol{x}_{ij}; \delta_i, \mu_i, \sigma_i^2)$, where δ_i and σ_i^2 are the parameters of interest and μ_i is the nuisance parameters. In order to construct the confidence intervals, the GPQs $\left(R(\boldsymbol{X}_{ij}; \boldsymbol{x}_{ij}, \delta_i, \mu_i, \sigma_i^2)\right)$, where $\boldsymbol{x}_{ij} = (x_{i1}, x_{i2}, \ldots, x_{in_i})$ is the observed sample, is not depend on the unknown parameters. Likewise, the observed value of $R\left(r(\boldsymbol{x}_{ij}; \boldsymbol{x}_{ij}, \delta_i, \mu_i, \sigma_i^2)\right)$ is not depend on the nuisance parameters. Then, $(R_{\alpha/2}, R_{1-\alpha/2})$ is the $100(1-\alpha)\%$ confidence interval for parameters of interest, where $R_{\alpha/2}$ and $R_{1-\alpha/2}$ be the $\alpha/2$ th and $(1 - \alpha/2)$ th percentiles of $R(\boldsymbol{X}_{ij}; \boldsymbol{x}_{ij}, \delta_i, \mu_i, \sigma_i^2)$. Since δ_i and σ_i^2 are the parameters of interest, therefore the GPQs for δ_i and σ_i^2 are desired.

Considering the GPQs for σ_i^2, Wu and Hsieh [10] used the idea of Krishnamoorthy and Mathew [27] to find the GPQs for σ_i^2 as follows [Equation (6)]

$$R_{\sigma_i^2}^{gci} = \frac{(n_{i(1)} - 1)s_i^2}{U_i} \tag{6}$$

where $U_i = (n_{i(1)} - 1)s_i^2 \big/ \sigma_i^2$ has chi-square distribution with $n_{i(1)} - 1$ degrees of freedom.

Subsequently, the GPQs for δ_i are applied with three concepts, such as VST, Wilson score method, and Jeffreys method as in the following.

2.1.1 The variance stabilizing transformation for the generalized confidence interval

To construct GPQ for δ_i, it uses VST to approximate the normal of the binomial distribution which was derived by Wu and Hsieh [10] as [Equation (7)]

$$R_{\delta_i}^{vst} = \sin^2\left[\arcsin\sqrt{\hat{\delta}_i} - \frac{1}{2\sqrt{n_i}} Z_i\right] \tag{7}$$

where $Z_i = 2\sqrt{n_i}\left(\arcsin\sqrt{\hat{\delta}_i} - \arcsin\sqrt{\delta_i}\right) \xrightarrow{D} N(0,1)$

as $n_i \to \infty$. Since GPQ for $R_{\sigma_i^2}^{gci}$ and $R_{\delta_i}^{vst}$ does not depend on the unknown parameters and the observed value of R does not depend on the nuisance parameters, the pivotal quantity for η_i is [Equation (8)]

$$R_{\eta_i}^{vst} = \sqrt{\frac{\exp\left(R_{\sigma_i^2}^{gci}\right) - R_{\delta_i}^{vst}}{R_{\delta_i}^{vst}}}. \tag{8}$$

By Equations (5) and (8), the pivotal quantity for ζ can be expressed by [Equation (9)]

$$R_{\zeta}^{vst} = \sqrt{\frac{\exp\left(R_{\sigma_1^2}^{gci}\right) - R_{\delta_1}^{vst}}{R_{\delta_1}^{vst}}} \Bigg/ \sqrt{\frac{\exp\left(R_{\sigma_2^2}^{gci}\right) - R_{\delta_2}^{vst}}{R_{\delta_2}^{vst}}}. \tag{9}$$

Therefore, the $100(1-\alpha)\%$ confidence interval for ζ using VST for GCI is [Equation (10)]

$$CI_{\zeta,(gci.vst)} = \left[R_{\zeta}^{vst}(\alpha/2), R_{\zeta}^{vst}(1-\alpha/2)\right], \tag{10}$$

where $R_{\zeta}^{vst}(\alpha/2)$ and $R_{\zeta}^{vst}(1-\alpha/2)$ are the $100(\alpha/2)$th and $100(1-\alpha/2)$th percentiles of the distribution of $R(\boldsymbol{X};\boldsymbol{x},\delta_i,\sigma_i^2)$, respectively.

2.1.2 The Wilson score method for the generalized confidence interval

For binomial distribution, Li *et al*. [8] obtained GPQ for δ_i by applying the score interval which was presented by Wilson [28] as follows [Equation (11)]

$$R_{\delta_i}^{w} = \frac{n_{i(1)} + \frac{Z_{iW}^2}{2}}{n_i + Z_{iW}^2} - \frac{Z_{iW}}{n_i + Z_{iW}^2}\sqrt{\frac{n_{i(0)}n_{i(1)}}{n_i} + \frac{Z_{iW}^2}{4}}, \tag{11}$$

where Z_{iw} has a standard normal distribution. It is seen that $R_{\delta_i}^{w}$ is an approximate GPQ because it does not depend on unknown parameters. Moreover, the observed value (see detail in [8]) does not depend on nuisance parameter. The pivotal quantity for η_i is [Equation (12)]

$$R_{\eta_i}^{w} = \sqrt{\frac{\exp\left(R_{\sigma_i^2}^{gci}\right) - R_{\delta_i}^{w}}{R_{\delta_i}^{w}}}. \tag{12}$$

The pivotal quantity for ζ is formed by using Equations (5) and (12) as

$$R_{\zeta}^{w} = \sqrt{\frac{\exp\left(R_{\sigma_1^2}^{gci}\right) - R_{\delta_1}^{w}}{R_{\delta_1}^{w}}} \Bigg/ \sqrt{\frac{\exp\left(R_{\sigma_2^2}^{gci}\right) - R_{\delta_2}^{w}}{R_{\delta_2}^{w}}}. \tag{13}$$

Therefore, the $100(1-\alpha)\%$ confidence interval for ζ using Wilson score method for GCI is [Equation (14)]

$$CI_{\zeta,(gci.w)} = \left[R_{\zeta}^{w}(\alpha/2), R_{\zeta}^{w}(1-\alpha/2)\right], \tag{14}$$

where $R_{\zeta}^{w}(\alpha/2)$ and $R_{\zeta}^{w}(1-\alpha/2)$ are the $100(\alpha/2)$th and $100(1-\alpha/2)$th percentiles of the distribution of $R(\boldsymbol{X};\boldsymbol{x},\delta_i,\sigma_i^2)$, respectively.

2.1.3 The Jeffreys method for the generalized confidence interval

Since $n_{i(0)}$ have a binomial distribution, the standard conjugate priors for these distributions are beta distributions [29]. Brown *et al*. [29] recommended the Jeffreys interval for the binomial proportion which uses the Jeffreys prior, $Beta(1/2,1/2)$. For GCI, Tian [12] suggested the GPQ for δ_i which uses beta variables as [Equation (15)]

$$R_{\delta_i}^{j}: Beta\left(n_{i(1)} + \frac{1}{2}, n_{i(0)} + \frac{1}{2}\right). \tag{15}$$

By Equations (4), (6), and (15), the pivotal quantity for η_i is [Equation (16)]

$$R_{\eta_i}^{j} = \sqrt{\frac{\exp\left(R_{\sigma_i^2}^{gci}\right) - R_{\delta_i}^{j}}{R_{\delta_i}^{j}}}. \tag{16}$$

Then, the pivotal quantity for ζ using Jeffreys method is expressed [Equation (17)]

$$R_{\zeta}^{j} = \sqrt{\frac{\exp\left(R_{\sigma_1^2}^{gci}\right) - R_{\delta_1}^{j}}{R_{\delta_1}^{j}}} \Bigg/ \sqrt{\frac{\exp\left(R_{\sigma_2^2}^{gci}\right) - R_{\delta_2}^{j}}{R_{\delta_2}^{j}}}. \tag{17}$$

Therefore, the $100(1-\alpha)\%$ confidence interval for ζ using Jeffreys method for GCI is [Equation (18)]

$$CI_{\zeta,(gci.j)} = \left[R_{\zeta}^{j}(\alpha/2), R_{\zeta}^{j}(1-\alpha/2)\right], \tag{18}$$

where $R_{\zeta}^{j}(\alpha/2)$ and $R_{\zeta}^{j}(1-\alpha/2)$ are the $100(\alpha/2)$th and $100(1-\alpha/2)$th percentiles of the distribution of $R(\boldsymbol{X};\boldsymbol{x},\delta_i,\sigma_i^2)$, respectively.

Algorithm 1
(For k = 1 to M)
- Generate $x_{ij},\ i=1,2,\ j=1,2,\ldots,n_i$ from $\Delta(\delta_i,\mu_i,\sigma_i^2)$;
- Compute $\hat{\delta}_i$ and s_i^2;

  (For l = 1 to m)
  - Generate Z_i from standard normal distribution;
  - Generate U_i from chi-square distribution with $n_{i(1)} - 1$ degrees of freedom;
  - Generate $Beta\left(n_{i(1)} + \frac{1}{2}, n_{i(0)} + \frac{1}{2}\right)$ from beta distribution;
  - Compute $R_{\sigma_i^2}^{gci}$ from Equation (6), R_{δ_i} from Equations (7), (11), and (15) and R_{ζ} from Equations (9), (13), and (17);

  (End l loop)
- Obtain an array of R_{ζ}'s;
- Compute the $R_{\zeta}(\alpha/2)$ and $R_{\zeta}(1-\alpha/2)$;

- If $R_{\zeta}(\alpha/2) \le \zeta \le R_{\zeta}(1-\alpha/2)$, then set $cp_k = 1$; else, set $cp_k = 0$ and compute $U_k - L_k$;

(End k loop)

- Compute the mean and standard deviation of the coverage probabilities;
- Compute the mean and standard deviation of the lengths.

2.2 *The method of variance estimates recovery*

To construct the confidence intervals for the functions of two parameters, MOVER introduced by Zou and Donner [30] and Zou *et al*. [31] is another method which can be used. This article focuses on the ratio of two independent coefficients of variation. The parameters of interest, η_1 and η_2, can be estimated by $\hat{\eta}_1$ and $\hat{\eta}_2$. The estimators s_i^2 and $\hat{\delta}_i$ will be substituted into Equation (4), as below [Equation (19)]

$$\hat{\eta}_1 = \sqrt{\frac{\exp\left(s_1^2\right)-\hat{\delta}_1}{\hat{\delta}_1}} \text{ and } \hat{\eta}_2 = \sqrt{\frac{\exp\left(s_2^2\right)-\hat{\delta}_2}{\hat{\delta}_2}}. \quad (19)$$

Thus, the $100(1-\alpha)\%$ two-sided confidence interval for ζ is given by [Equation (20)]

$$CI_{\zeta} = \left[L_m, U_m\right] \quad (20)$$

According to Donner and Zou [13], the lower and the upper bounds can be applied as follows

$$L_m = \frac{\hat{\eta}_1\hat{\eta}_2 - \sqrt{\left(\hat{\eta}_1\hat{\eta}_2\right)^2 - l_1u_2\left(2\hat{\eta}_1 - l_1\right)\left(2\hat{\eta}_2 - u_2\right)}}{u_2\left(2\hat{\eta}_2 - u_2\right)} \quad (21)$$

and

$$U_m = \frac{\hat{\eta}_1\hat{\eta}_2 + \sqrt{\left(\hat{\eta}_1\hat{\eta}_2\right)^2 - u_1l_2\left(2\hat{\eta}_1 - u_1\right)\left(2\hat{\eta}_2 - l_2\right)}}{l_2\left(2\hat{\eta}_2 - l_2\right)}. \quad (22)$$

By the coefficients of variation, the confidence limits for σ_i^2 and δ_i are used in Equations (21) and (22). Firstly, consider the confidence intervals for σ_i^2. Since the unbiased estimator for σ_i^2 is [Equation (23)]

$$s_i^2 = \frac{1}{n_{i(1)}-1}\sum_{j=1}^{n_{i(1)}}\left(Y_{ij} - \bar{Y}_i\right)^2, \quad (23)$$

where $\left(n_{i(1)}-1\right)s_i^2 / \sigma_i^2$ is a chi-square distribution with $n_{i(1)}-1$ degrees of freedom. The coverage probability of it at the significance level α is [Equation (24)]

$$P\left(\chi^2_{\frac{\alpha}{2}, n_{i(1)}-1} \le \frac{\left(n_{i(1)}-1\right)s_i^2}{\sigma_i^2} \le \chi^2_{1-\frac{\alpha}{2}, n_{i(1)}-1}\right) = 1-\alpha \quad (24)$$

Thus, the lower and the upper bounds for σ_i^2 can be expressed as [Equation (25)]

$$\frac{\left(n_{i(1)}-1\right)s_i^2}{\chi^2_{1-\frac{\alpha}{2}, n_{i(1)}-1}} \le \sigma_i^2 \le \frac{\left(n_{i(1)}-1\right)s_i^2}{\chi^2_{\frac{\alpha}{2}, n_{i(1)}-1}} \quad (25)$$

2.2.1 The variance stabilizing transformation for the method of variance estimates recovery

The VST was presented by DasGupta [32], this used the delta method for construction. The sample size $n_{i(0)}$ has a binomial distribution with the proportion of zero values $1-\delta_i$. By using the delta method, Wu and Hsieh [10] obtained VST of a binomial distribution which approximated normal distribution to be the arcsine square-root transformation. For parameter δ_i and samples size n_i, VST is $\arcsin\sqrt{\delta_i}$ (see details in [32] and [10]). By Guan [33], $\sqrt{n_i}\left(\arcsin\sqrt{\hat{\delta}_i} - \arcsin\sqrt{\delta_i}\right) \xrightarrow{D} N\left(0, 1/4\right)$.

Then, $Z_i = 2\sqrt{n_i}\left(\arcsin\sqrt{\hat{\delta}_i} - \arcsin\sqrt{\delta_i}\right) \xrightarrow{D} N\left(0,1\right)$ as $n_i \to \infty$. Hence, the $100(1-\alpha)\%$ asymptotically confidence interval for δ_i is [Equation (26) and (27)]

$$CI_{\delta_i}^{vst} = \sin^2\left(\arcsin\sqrt{\hat{\delta}_i} \pm \frac{1}{2\sqrt{n_i}} Z_{i\left(1-\frac{\alpha}{2}\right)}\right). \quad (26)$$

Let

$$l_i^{vst} = \sqrt{\frac{\exp\left(l_{\sigma_i^2}\right) - u_{\delta_i}^{vst}}{u_{\delta_i}^{vst}}} \text{ and } u_i^{vst} = \sqrt{\frac{\exp\left(u_{\sigma_i^2}\right) - l_{\delta_i}^{vst}}{l_{\delta_i}^{vst}}}. \quad (27)$$

Therefore, the $100(1-\alpha)\%$ confidence interval for δ_i using VST is [Equation (28)]

$$CI_{\zeta,(m.vst)} = \left[L_{m.vst}, U_{m.vst}\right], \quad (28)$$

where

$$L_{m.vst} = \frac{\hat{\eta}_1\hat{\eta}_2 - \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - l_1^{vst}u_2^{vst}(2\hat{\eta}_1 - l_1^{vst})(2\hat{\eta}_2 - u_2^{vst})}}{u_2^{vst}(2\hat{\eta}_2 - u_2^{vst})}$$

and

$$U_{m.vst} = \frac{\hat{\eta}_1\hat{\eta}_2 + \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - u_1^{vst}l_2^{vst}(2\hat{\eta}_1 - u_1^{vst})(2\hat{\eta}_2 - l_2^{vst})}}{l_2^{vst}(2\hat{\eta}_2 - l_2^{vst})}.$$

2.2.2 The Wilson score method for the method of variance estimates recovery

The Wilson score method was first presented by Wilson [28]. To construct the confidence interval, the limits of this method can be obtained by the score method. For binomial proportions, Brown *et al*. [29] and Donner and Zou [34] established the confidence interval using the Wilson score method. Thus, the confidence interval for δ_i is [Equation (29) and (30)]

$$CI_{\delta_i}^{w} = \frac{n_{i(1)} + \frac{Z_{i(\alpha/2)}^2}{2}}{n_i + Z_{i(\alpha/2)}^2} \pm Z_{i(\alpha/2)} \frac{\sqrt{\frac{n_{i(0)}n_{i(1)}}{n_i} + \frac{Z_{i(\alpha/2)}^2}{4}}}{n_i + Z_{i(\alpha/2)}^2}. \quad (29)$$

Let

$$l_i^w = \sqrt{\frac{\exp(l_{\sigma_i^2}) - u_{\delta_i}^w}{u_{\delta_i}^w}} \text{ and } u_i^w = \sqrt{\frac{\exp(u_{\sigma_i^2}) - l_{\delta_i}^w}{l_{\delta_i}^w}}. \quad (30)$$

Therefore, the $100(1-\alpha)\%$ confidence interval for ζ using Wilson score method is [Equation (31)]

$$CI_{\zeta,(m.w)} = [L_{m.w}, U_{m.w}], \quad (31)$$

where

$$L_{m.w} = \frac{\hat{\eta}_1\hat{\eta}_2 - \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - l_1^{w}u_2^{w}(2\hat{\eta}_1 - l_1^{w})(2\hat{\eta}_2 - u_2^{w})}}{u_2^{w}(2\hat{\eta}_2 - u_2^{w})}$$

and

$$U_{m.w} = \frac{\hat{\eta}_1\hat{\eta}_2 + \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - u_1^{w}l_2^{w}(2\hat{\eta}_1 - u_1^{w})(2\hat{\eta}_2 - l_2^{w})}}{l_2^{w}(2\hat{\eta}_2 - l_2^{w})}.$$

2.2.3 The Jeffreys method for the method of variance estimates recovery

Brown *et al*. [29] recommended Jeffreys method which had the alternative intervals construction for binomial distribution. It used beta prior for binomial proportion [35]. Given a prior and a posterior for δ_i as $Beta(a,b)$ and $Beta(n_{i(1)}+a, n_{i(0)}+b)$, respectively. In this article, one set $Beta(1/2,1/2)$, following Blom [36], is the Jeffreys prior. Jeffreys prior limits for δ_i is [Equations (32) and (33)]

$$CI_{\delta_i}^{j} = \left[Beta\left(\frac{\alpha}{2}; n_{i(1)} + \frac{1}{2}, n_{i(0)} + \frac{1}{2}\right), Beta\left(1-\frac{\alpha}{2}; n_{i(1)} + \frac{1}{2}, n_{i(0)} + \frac{1}{2}\right)\right]. \quad (32)$$

Let

$$l_i^j = \sqrt{\frac{\exp(l_{\sigma_i^2}) - u_{\delta_i}^j}{u_{\delta_i}^j}} \text{ and } u_i^j = \sqrt{\frac{\exp(u_{\sigma_i^2}) - l_{\delta_i}^j}{l_{\delta_i}^j}}. \quad (33)$$

Therefore, the $100(1-\alpha)\%$ confidence interval for ζ using Jeffreys method is [Equation (34)]

$$CI_{\zeta,(m.j)} = [L_{m.j}, U_{m.j}], \quad (34)$$

where

$$L_{m.j} = \frac{\hat{\eta}_1\hat{\eta}_2 - \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - l_1^{j}u_2^{j}(2\hat{\eta}_1 - l_1^{j})(2\hat{\eta}_2 - u_2^{j})}}{u_2^{j}(2\hat{\eta}_2 - u_2^{j})}$$

and

$$U_{m.j} = \frac{\hat{\eta}_1\hat{\eta}_2 + \sqrt{(\hat{\eta}_1\hat{\eta}_2)^2 - u_1^{j}l_2^{j}(2\hat{\eta}_1 - u_1^{j})(2\hat{\eta}_2 - l_2^{j})}}{l_2^{j}(2\hat{\eta}_2 - l_2^{j})}.$$

Algorithm 2
(For $k = 1$ to M)
- Generate x_{ij}, $i = 1,2,\ j = 1,2,\ldots,n_i$ from $\Delta(\delta_i, \mu_i, \sigma_i^2)$;
- Compute $\hat{\delta}_i$ and s_i^2;
- Generate Z_i from standard normal distribution;
- Generate $Beta\left(n_{i(1)}+\frac{1}{2}, n_{i(0)}+\frac{1}{2}\right)$ from beta distribution;

- Compute l_i and u_i from Equations (27), (30), and (33);
- Compute the 95% confidence intervals for ζ from Equations (28), (31), and (34);
- If $L \le \zeta \le U$, then set $cp_k = 1$; else, set $cp_k = 0$ and compute $U_k - L_k$;

(End k loop)

- Compute the mean and standard deviation of the coverage probabilities;
- Compute the mean and standard deviation of the lengths.

3 Results

3.1 *Simulation study*

The performances of the confidence intervals for the ratio of the independent coefficients of variation of two delta-lognormal distributions were compared. For this simulation, random samples were used to generate 15,000 sets from a delta-lognormal distribution and 5,000 pivotal quantities for GCI with combinations of sample sizes $n_1, n_2 = 25, 50, 100$; $\delta_1, \delta_2 = 0.2, 0.5, 0.8$; and $\sigma_1^2, \sigma_2^2 = 0.5, 1.0, 2.0$. For this study, the cases with an expected non-zero value of less than 10 were discarded, thereby following the methodology of Fletcher [9] and Wu and Hsieh [10]. Coverage probabilities and expected lengths were used to evaluate the performances of the proposed confidence intervals based on the coverage probability closest to the nominal confidence level of 0.95 and with the shortest expected length.

The results in Tables 1 and 2 reveal that GCI outperformed the MOVER methods due to the coverage probabilities of the GCI methods being closest to the nominal level except for the cases where $\delta_1, \delta_2 = 0.2, 0.5$ together with $\sigma_1^2 : \sigma_2^2 = 0.5{:}0.5$ or $0.5{:}1.0$. Moreover, the expected lengths were short in nearly every case. In addition, the performance of GCI based on VST ($CI_{gci.vst}$) is appropriate for unequal sample sizes with $\delta_1, \delta_2 = 0.8$, while GCI based on the Wilson score method ($CI_{gci.w}$) performed the best for both equal and unequal sample sizes together with $\delta_1, \delta_2 = 0.2, 0.5$. GCI based on Jeffrey's method ($CI_{gci.j}$) is recommended in cases of equal sample sizes and $\delta_1, \delta_2 = 0.8$. Moreover, the MOVER methods based on VST ($CI_{gci.vst}$), Wilson score method ($CI_{m.w}$), and Jeffreys method ($CI_{m.j}$) had coverage probabilities close to 1 in almost all cases. It is notable that the expected lengths in cases with equal variances were longer than cases with unequal variances.

3.2 *An empirical study*

Rainfall data series from two rain stations at Ban Thap Tawi Water Supply and Ban Lert Sawat School, Sikhiu District, Nakhon Ratchasima, Thailand, collected by the Lower Northeastern Region Hydrological Irrigation Center, were used to illustrate the efficacy of the methods used to establish confidence intervals in this article (the datasets are reported in Table 3). These were used because there is often flooding in this area during the rainy season, and thus it is imperative to monitor rainfall amounts to mitigate adverse effects due to this. Because the dispersion of rain can usually be clarified using the coefficient of variation, statistical inference based on this can be used on rainfall data to aid planning to cope with repeated flooding.

To analyze the data, we first considered the dispersion of both datasets (shown as histograms in Figure 1), which revealed that the positive observations for each area are right-skewed. The minimum Akaike information criterion (AIC) was used to analyze their distributions. The results in Table 4 indicate that the distributions of the positive values from both areas are lognormal since their AIC values were less than for other distributions. Moreover, normal Q-Q plots of the log-transformed data series presented in Figure 2 confirm the minimum AIC analysis. Since the rainfall series from both areas include zero observations with binomial distributions, the two datasets follow delta-lognormal distributions. The statistical summary of the rainfall data from the Ban Thap Tawi Water Supply station is $n_1 = 72$, $\delta_1 = 0.8472$, $\hat{\mu}_1 = 4.1400$, $s_1^2 = 1.3737$, and $\hat{\eta}_1 = 0.2831$ and that from the Ban Lert Sawat School station is $n_2 = 72$, $\delta_2 = 0.8056$, $\hat{\mu}_2 = 4.1198$, $s_2^2 = 1.2783$, and $\hat{\eta}_2 = 0.2744$. The ratio of $\hat{\eta}_1$ and $\hat{\eta}_2$ is $\zeta = 1.0317$. The 95% confidence intervals for ζ are reported in Table 5. These results indicate that the expected lengths for GCI were shorter than those of the MOVER methods, and thus GCI can be used to construct confidence intervals for the ratio of the independent coefficients of variation of these two rainfall series.

Table 1: The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the ratio of two independent coefficients of variation of the delta-lognormal distribution: equal sample sizes

$n_1 : n_2$	$\delta_1 : \delta_2$	$\sigma_1^2 : \sigma_2^2$	Coverage Probabilities (Standard Deviation)						Expected Lengths (Standard Deviation)					
			$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$	$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$
25:25	0.5:0.5	0.5:0.5	0.9827	0.9824	0.9824	0.9986	0.9986	0.9973	1.4678	*1.4468*	1.4559	2.1792	2.1312	2.0195
			(0.1305)	(0.1315)	(0.1315)	(0.0374)	(0.0374)	(0.0516)	(0.5643)	(0.5595)	(0.5604)	(0.8431)	(0.8231)	(0.8007)
		0.5:1.0	0.9728	0.9721	0.9724	0.9964	0.9963	0.9936	1.1858	*1.1750*	1.1798	1.7094	1.6789	1.6050
			(0.1627)	(0.1646)	(0.1638)	(0.0599)	(0.0610)	(0.0797)	(0.4291)	(0.4255)	(0.4266)	(0.6637)	(0.6491)	(0.6328)
		0.5:2.0	0.9623	0.9615	0.9620	0.9886	0.9876	0.9843	0.8673	*0.8626*	0.8645	1.1718	1.1551	1.1125
			(0.1906)	(0.1925)	(0.1912)	(0.1062)	(0.1107)	(0.1242)	(0.3398)	(0.3371)	(0.3383)	(0.5231)	(0.5130)	(0.4963)
		1.0:1.0	0.9687	0.9679	0.9687	0.9943	0.9938	0.9921	2.6845	*2.6718*	2.6756	3.8783	3.8220	3.6700
			(0.1742)	(0.1764)	(0.1740)	(0.0755)	(0.0785)	(0.0887)	(2.3226)	(2.3161)	(2.3144)	(3.3235)	(3.2732)	(3.1509)
		1.0:2.0	0.9587	0.9585	0.9584	0.9875	0.9871	0.9841	1.7778	*1.7722*	1.7749	2.5063	2.4752	2.3872
			(0.1989)	(0.1995)	(0.1997)	(0.1110)	(0.1127)	(0.1250)	(1.4730)	(1.4695)	(1.4673)	(2.1710)	(2.1402)	(2.0647)
		2.0:2.0	0.9578	0.9574	0.9578	0.9845	0.9839	0.9811	10.7520	*10.7360*	10.7481	15.9337	15.7511	15.2480
			(0.2011)	(0.2020)	(0.2011)	(0.1234)	(0.1257)	(0.1363)	(35.3567)	(35.3057)	(35.5486)	(55.3430)	(54.5906)	(52.7463)
	0.8:0.8	0.5:0.5	0.9669	0.9695	0.9683	0.9949	0.9962	0.9912	1.2300	1.2290	*1.2227*	1.7000	1.7031	1.6167
			(0.1788)	(0.1721)	(0.1753)	(0.0715)	(0.0615)	(0.0934)	(0.3735)	(0.3695)	(0.3687)	(0.5115)	(0.5060)	(0.4922)
		0.5:1.0	0.9578	0.9601	0.9592	0.9903	0.9918	0.9885	0.9076	0.9100	*0.9064*	1.1917	1.2021	1.1497
			(0.2011)	(0.1958)	(0.1978)	(0.0982)	(0.0902)	(0.1065)	(0.2703)	(0.2684)	(0.2678)	(0.3717)	(0.3713)	(0.3592)
		0.5:2.0	0.9551	0.9566	0.9558	0.9803	0.9814	0.9770	0.6239	0.6264	*0.6249*	0.7620	0.7697	0.7428
			(0.2072)	(0.2038)	(0.2056)	(0.1391)	(0.1351)	(0.1499)	(0.1948)	(0.1942)	(0.1942)	(0.2628)	(0.2645)	(0.2556)
		1.0:1.0	0.9553	0.9564	0.9552	0.9859	0.9872	0.9831	1.8278	1.8265	*1.8231*	2.3478	2.3688	2.2830
			(0.2067)	(0.2042)	(0.2069)	(0.1178)	(0.1124)	(0.1290)	(0.9088)	(0.9057)	(0.9046)	(1.1561)	(1.1631)	(1.1302)
		1.0:2.0	0.9554	0.9559	0.9561	0.9781	0.9796	0.9762	1.1795	1.1804	*1.1786*	1.4490	1.4636	1.4188
			(0.2064)	(0.2053)	(0.2050)	(0.1465)	(0.1414)	(0.1524)	(0.5979)	(0.5967)	(0.5950)	(0.7646)	(0.7718)	(0.7481)
		2.0:2.0	0.9515	0.9519	0.9517	0.9743	0.9753	0.9711	4.1834	4.1824	*4.1795*	5.2196	5.2784	5.1332
			(0.2149)	(0.2139)	(0.2143)	(0.1583)	(0.1551)	(0.1676)	(4.7046)	(4.7011)	(4.6921)	(5.9701)	(6.0325)	(5.8766)
50:50	0.2:0.2	0.5:0.5	0.9874	0.9867	0.9875	0.9996	0.9995	0.9991	1.6028	*1.5717*	1.5851	2.4656	2.3685	2.3132
			(0.1115)	(0.1144)	(0.1113)	(0.0200)	(0.0216)	(0.0305)	(0.8923)	(0.8869)	(0.8933)	(1.4225)	(1.3552)	(1.3476)
		0.5:1.0	0.9797	0.9776	0.9788	0.9980	0.9980	0.9969	1.3757	*1.3542*	1.3632	2.0805	2.0087	1.9637
			(0.1409)	(0.1480)	(0.1441)	(0.0447)	(0.0447)	(0.0559)	(0.6645)	(0.6587)	(0.6600)	(1.1015)	(1.0484)	(1.0400)
		0.5:2.0	0.9642	0.9635	0.9639	0.9917	0.9914	0.9892	1.0495	*1.0372*	1.0421	1.5137	1.4714	1.4407
			(0.1858)	(0.1875)	(0.1866)	(0.0909)	(0.0923)	(0.1034)	(0.4793)	(0.4737)	(0.4754)	(0.8158)	(0.7778)	(0.7729)
		1.0:1.0	0.9763	0.9753	0.9765	0.9971	0.9967	0.9957	3.7055	*3.6816*	3.6934	5.8088	5.5969	5.4993
			(0.1520)	(0.1551)	(0.1516)	(0.0535)	(0.0571)	(0.0652)	(6.0543)	(6.0490)	(6.1052)	(9.6992)	(9.2973)	(9.2472)
		1.0:2.0	0.9650	0.9642	0.9645	0.9929	0.9923	0.9918	2.4756	*2.4612*	2.4687	3.9099	3.7789	3.7179
			(0.1838)	(0.1858)	(0.1851)	(0.0842)	(0.0872)	(0.0902)	(3.7239)	(3.7266)	(3.8121)	(6.4683)	(6.1973)	(6.2154)
50:50	0.2:0.2	2.0:2.0	0.9606	0.9598	0.9605	0.9903	0.9893	0.9875	37.1133	37.0390	36.8115	64.7610	62.2522	61.5205
			(0.1946)	(0.1964)	(0.1949)	(0.0982)	(0.1031)	(0.1110)	(387.1329)	(385.2160)	(367.1065)	(677.2387)	(648.3719)	(644.1015)
	0.5:0.5	0.5:0.5	0.9820	0.9818	0.9822	0.9985	0.9985	0.9975	0.8895	*0.8820*	0.8856	1.2862	1.2714	1.2314
			(0.1330)	(0.1337)	(0.1322)	(0.0383)	(0.0383)	(0.0496)	(0.1917)	(0.1904)	(0.1911)	(0.2781)	(0.2743)	(0.2745)
		0.5:1.0	0.9727	0.9725	0.9721	0.9963	0.9962	0.9951	0.7391	*0.7353*	0.7371	1.0310	1.0215	0.9946
			(0.1629)	(0.1634)	(0.1648)	(0.0610)	(0.0615)	(0.0696)	(0.1485)	(0.1474)	(0.1481)	(0.2240)	(0.2211)	(0.2195)
		0.5:2.0	0.9561	0.9559	0.9557	0.9867	0.9865	0.9849	0.5635	*0.5620*	0.5628	0.7254	0.7204	0.7059
			(0.2050)	(0.2053)	(0.2058)	(0.1144)	(0.1156)	(0.1221)	(0.1230)	(0.1223)	(0.1227)	(0.1835)	(0.1815)	(0.1781)
		1.0:1.0	0.9647	0.9643	0.9640	0.9942	0.9942	0.9926	1.3351	*1.3310*	1.3327	1.8414	1.8278	1.7871
			(0.1845)	(0.1856)	(0.1863)	(0.0759)	(0.0759)	(0.0857)	(0.5140)	(0.5130)	(0.5134)	(0.6912)	(0.6860)	(0.6772)
		1.0:2.0	0.9575	0.9571	0.9570	0.9881	0.9878	0.9857	0.9514	*0.9497*	0.9506	1.2456	1.2381	1.2151
			(0.2017)	(0.2027)	(0.2029)	(0.1083)	(0.1098)	(0.1189)	(0.3452)	(0.3445)	(0.3447)	(0.4759)	(0.4725)	(0.4659)
		2.0:2.0	0.9555	0.9553	0.9553	0.9853	0.9851	0.9838	2.8154	*2.8129*	2.8133	3.6851	3.6673	3.6080
			(0.2063)	(0.2067)	(0.2067)	(0.1205)	(0.1213)	(0.1263)	(2.1458)	(2.1437)	(2.1400)	(2.8117)	(2.7970)	(2.7604)

Table 1: The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the ratio of two independent coefficients of variation of the delta-lognormal distribution: equal sample sizes (*Continued*)

$n_1 : n_2$	$\delta_1 : \delta_2$	$\sigma_1^2 : \sigma_2^2$	Coverage Probabilities (Standard Deviation)						Expected Lengths (Standard Deviation)					
			$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$	$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$
	0.8:0.8	0.5:0.5	0.9675	0.9687	0.9683	0.9955	0.9956	0.9942	0.7793	0.7791	0.7771	1.0721	1.0713	1.0392
			(0.1774)	(0.1742)	(0.1753)	(0.0672)	(0.0662)	(0.0759)	(0.1460)	(0.1451)	(0.1454)	(0.1978)	(0.1962)	(0.1946)
		0.5:1.0	0.9606	0.9620	0.9622	0.9911	0.9914	0.9895	0.5831	0.5841	0.5828	0.7584	0.7611	0.7416
			(0.1946)	(0.1912)	(0.1907)	(0.0941)	(0.0923)	(0.1018)	(0.1069)	(0.1064)	(0.1063)	(0.1458)	(0.1454)	(0.1432)
		0.5:2.0	0.9527	0.9531	0.9525	0.9805	0.9807	0.9786	0.4122	0.4132	0.4127	0.4954	0.4974	0.4881
			(0.2122)	(0.2115)	(0.2126)	(0.1384)	(0.1375)	(0.1447)	(0.0818)	(0.0817)	(0.0817)	(0.1085)	(0.1089)	(0.1069)
		1.0:1.0	0.9551	0.9558	0.9553	0.9869	0.9876	0.9855	1.0608	1.0607	1.0596	1.3367	1.3418	1.3138
			(0.2070)	(0.2056)	(0.2067)	(0.1139)	(0.1107)	(0.1197)	(0.2978)	(0.2973)	(0.2971)	(0.3683)	(0.3693)	(0.3641)
		1.0:2.0	0.9539	0.9544	0.9541	0.9803	0.9811	0.9783	0.7098	0.7103	0.7099	0.8469	0.8505	0.8365
			(0.2098)	(0.2086)	(0.2092)	(0.1391)	(0.1361)	(0.1456)	(0.1975)	(0.1974)	(0.1971)	(0.2448)	(0.2460)	(0.2422)
		2.0:2.0	0.9527	0.9530	0.9527	0.9759	0.9765	0.9748	1.8793	1.8792	1.8788	2.2189	2.2304	2.1980
			(0.2124)	(0.2117)	(0.2124)	(0.1535)	(0.1516)	(0.1567)	(0.9356)	(0.9351)	(0.9355)	(1.1039)	(1.1101)	(1.0972)
100:100	0.2:0.2	0.5:0.5	0.9877	0.9875	0.9873	0.9997	0.9997	0.9993	0.8938	0.8829	0.8876	1.3078	1.2819	1.2646
			(0.1101)	(0.1113)	(0.1118)	(0.0183)	(0.0183)	(0.0258)	(0.2057)	(0.2042)	(0.2047)	(0.3154)	(0.3068)	(0.3102)
		0.5:1.0	0.9811	0.9799	0.9796	0.9981	0.9980	0.9973	0.8030	0.7960	0.7990	1.1473	1.1290	1.1160
			(0.1363)	(0.1405)	(0.1414)	(0.0439)	(0.0447)	(0.0516)	(0.1642)	(0.1626)	(0.1633)	(0.2616)	(0.2541)	(0.2553)
		0.5:2.0	0.9617	0.9611	0.9613	0.9913	0.9909	0.9903	0.6575	0.6539	0.6552	0.8739	0.8649	0.8563
			(0.1919)	(0.1933)	(0.1930)	(0.0930)	(0.0951)	(0.0982)	(0.1424)	(0.1410)	(0.1413)	(0.2250)	(0.2192)	(0.2191)
		1.0:1.0	0.9729	0.9723	0.9728	0.9975	0.9975	0.9968	1.4543	1.4473	1.4506	2.1010	2.0658	2.0486
			(0.1625)	(0.1642)	(0.1627)	(0.0496)	(0.0503)	(0.0565)	(0.6309)	(0.6296)	(0.6307)	(0.8972)	(0.8791)	(0.8800)
		1.0:2.0	0.9601	0.9595	0.9598	0.9915	0.9911	0.9901	1.0916	1.0878	1.0895	1.5094	1.4895	1.4772
			(0.1958)	(0.1971)	(0.1964)	(0.0920)	(0.0941)	(0.0988)	(0.4677)	(0.4665)	(0.4666)	(0.6880)	(0.6742)	(0.6706)
		2.0:2.0	0.9589	0.9582	0.9583	0.9899	0.9893	0.9882	3.5359	3.5300	3.5337	4.9568	4.8828	4.8587
			(0.1986)	(0.2001)	(0.1998)	(0.1002)	(0.1027)	(0.1080)	(3.5260)	(3.5212)	(3.5237)	(4.9298)	(4.8464)	(4.8429)
	0.5:0.5	0.5:0.5	0.9807	0.9804	0.9805	0.9985	0.9985	0.9977	0.5877	0.5850	0.5863	0.8402	0.8353	0.8211
			(0.1375)	(0.1386)	(0.1382)	(0.0383)	(0.0391)	(0.0476)	(0.0803)	(0.0800)	(0.0802)	(0.1148)	(0.1140)	(0.1160)
		0.5:1.0	0.9700	0.9695	0.9699	0.9959	0.9958	0.9952	0.4944	0.4931	0.4936	0.6815	0.6783	0.6691
			(0.1706)	(0.1719)	(0.1708)	(0.0642)	(0.0647)	(0.0691)	(0.0635)	(0.0632)	(0.0632)	(0.0943)	(0.0936)	(0.0941)
		0.5:2.0	0.9565	0.9563	0.9559	0.9876	0.9873	0.9859	0.3863	0.3858	0.3860	0.4880	0.4864	0.4819
			(0.2041)	(0.2044)	(0.2054)	(0.1107)	(0.1118)	(0.1178)	(0.0544)	(0.0542)	(0.0543)	(0.0801)	(0.0797)	(0.0794)
		1.0:1.0	0.9627	0.9624	0.9624	0.9947	0.9946	0.9933	0.8330	0.8316	0.8324	1.1293	1.1251	1.1118
			(0.1896)	(0.1902)	(0.1902)	(0.0728)	(0.0733)	(0.0814)	(0.1884)	(0.1882)	(0.1884)	(0.2480)	(0.2470)	(0.2461)
		1.0:2.0	0.9561	0.9559	0.9561	0.9888	0.9886	0.9873	0.6104	0.6099	0.6102	0.7772	0.7749	0.7678
			(0.2048)	(0.2053)	(0.2048)	(0.1052)	(0.1062)	(0.1118)	(0.1318)	(0.1317)	(0.1316)	(0.1774)	(0.1768)	(0.1754)
		2.0:2.0	0.9533	0.9531	0.9530	0.9841	0.9839	0.9825	1.5022	1.5015	1.5014	1.8804	1.8761	1.8618
			(0.2111)	(0.2114)	(0.2117)	(0.1250)	(0.1257)	(0.1313)	(0.6121)	(0.6118)	(0.6113)	(0.7571)	(0.7554)	(0.7516)
	0.8:0.8	0.5:0.5	0.9660	0.9668	0.9663	0.9958	0.9958	0.9948	0.5251	0.5251	0.5244	0.7217	0.7211	0.7098
			(0.1812)	(0.1792)	(0.1806)	(0.0647)	(0.0647)	(0.0719)	(0.0671)	(0.0668)	(0.0668)	(0.0906)	(0.0901)	(0.0904)
		0.5:1.0	0.9613	0.9622	0.9608	0.9927	0.9929	0.9917	0.3970	0.3974	0.3969	0.5151	0.5158	0.5089
			(0.1930)	(0.1907)	(0.1941)	(0.0853)	(0.0838)	(0.0906)	(0.0489)	(0.0488)	(0.0489)	(0.0664)	(0.0663)	(0.0661)
		0.5:2.0	0.9507	0.9509	0.9511	0.9795	0.9799	0.9787	0.2842	0.2845	0.2843	0.3396	0.3402	0.3370
			(0.2166)	(0.2160)	(0.2156)	(0.1416)	(0.1402)	(0.1445)	(0.0377)	(0.0377)	(0.0377)	(0.0499)	(0.0499)	(0.0496)
		1.0:1.0	0.9562	0.9565	0.9571	0.9869	0.9871	0.9862	0.6951	0.6951	0.6947	0.8707	0.8721	0.8627
			(0.2047)	(0.2041)	(0.2027)	(0.1136)	(0.1130)	(0.1167)	(0.1269)	(0.1267)	(0.1267)	(0.1555)	(0.1557)	(0.1546)
		1.0:2.0	0.9528	0.9531	0.9528	0.9798	0.9799	0.9791	0.4730	0.4731	0.4730	0.5586	0.5597	0.5551
			(0.2121)	(0.2114)	(0.2121)	(0.1407)	(0.1402)	(0.1432)	(0.0855)	(0.0854)	(0.0855)	(0.1046)	(0.1049)	(0.1041)
		2.0:2.0	0.9519	0.9518	0.9519	0.9755	0.9759	0.9745	1.1322	1.1322	1.1320	1.3104	1.3136	1.3044
			(0.2141)	(0.2142)	(0.2141)	(0.1547)	(0.1533)	(0.1575)	(0.3370)	(0.3369)	(0.3369)	(0.3875)	(0.3886)	(0.3862)

Table 2: The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the ratio of two independent coefficients of variation of the delta-lognormal distribution: unequal sample sizes

$n_1 : n_2$	$\delta_1 : \delta_2$	$\sigma_1^2 : \sigma_2^2$	Coverage Probabilities (Standard Deviation)						Expected Lengths (Standard Deviation)					
			$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$	$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$
25:50	0.5:0.5	0.5:0.5	0.9814	0.9811	0.9812	0.9983	0.9983	0.9965	1.2421	1.2290	1.2357	1.8263	1.7984	1.7139
			(0.1351)	(0.1361)	(0.1358)	(0.0416)	(0.0416)	(0.0593)	(0.5064)	(0.5050)	(0.5065)	(0.7251)	(0.7133)	(0.6983)
		0.5:1.0	0.9745	0.9737	0.9748	0.9968	0.9964	0.9949	0.9742	0.9668	0.9706	1.4111	1.3929	1.3345
			(0.1575)	(0.1599)	(0.1567)	(0.0565)	(0.0599)	(0.0710)	(0.3769)	(0.3756)	(0.3755)	(0.5531)	(0.5444)	(0.5347)
		0.5:2.0	0.9597	0.9589	0.9595	0.9895	0.9895	0.9873	0.6883	0.6851	0.6867	0.9431	0.9332	0.9012
			(0.1966)	(0.1985)	(0.1971)	(0.1018)	(0.1021)	(0.1118)	(0.2633)	(0.2621)	(0.2622)	(0.3928)	(0.3871)	(0.3775)
		1.0:1.0	0.9681	0.9674	0.9685	0.9943	0.9943	0.9917	2.3662	2.3595	2.3630	3.3301	3.2958	3.1798
			(0.1758)	(0.1776)	(0.1748)	(0.0755)	(0.0751)	(0.0906)	(2.2065)	(2.2022)	(2.1953)	(3.0279)	(2.9925)	(2.9223)
		1.0:2.0	0.9599	0.9595	0.9595	0.9883	0.9880	0.9863	1.5093	1.5063	1.5085	2.1013	2.0823	2.0147
			(0.1963)	(0.1972)	(0.1971)	(0.1074)	(0.1089)	(0.1164)	(1.3706)	(1.3690)	(1.3686)	(1.9593)	(1.9379)	(1.8986)
		2.0:2.0	0.9553	0.9551	0.9559	0.9840	0.9835	0.9812	9.9256	9.9171	9.9180	13.7743	13.6543	13.2521
			(0.2066)	(0.2070)	(0.2053)	(0.1255)	(0.1275)	(0.1358)	(31.8452)	(31.7701)	(31.6338)	(44.2537)	(43.7562)	(42.3321)
	0.8:0.8	0.5:0.5	0.9691	0.9716	0.9709	0.9947	0.9955	0.9927	1.0500	1.0542	1.0500	1.4391	1.4562	1.3858
			(0.1730)	(0.1661)	(0.1682)	(0.0728)	(0.0672)	(0.0849)	(0.3129)	(0.3109)	(0.3114)	(0.4070)	(0.4072)	(0.4008)
		0.5:1.0	0.9607	0.9628	0.9627	0.9902	0.9914	0.9879	0.7537	0.7580	0.7554	0.9990	1.0143	0.9689
			(0.1944)	(0.1893)	(0.1894)	(0.0985)	(0.0923)	(0.1092)	(0.2265)	(0.2255)	(0.2258)	(0.2958)	(0.2970)	(0.2910)
		0.5:2.0	0.9553	0.9570	0.9560	0.9847	0.9853	0.9816	0.4925	0.4957	0.4946	0.6162	0.6259	0.6027
			(0.2066)	(0.2029)	(0.2051)	(0.1229)	(0.1202)	(0.1344)	(0.1466)	(0.1463)	(0.1465)	(0.1924)	(0.1941)	(0.1891)
		1.0:1.0	0.9549	0.9554	0.9552	0.9859	0.9870	0.9833	1.6030	1.6062	1.6047	2.0297	2.0590	1.9868
			(0.2075)	(0.2064)	(0.2069)	(0.1178)	(0.1133)	(0.1283)	(0.8399)	(0.8398)	(0.8406)	(1.0221)	(1.0331)	(1.0103)
		1.0:2.0	0.9531	0.9539	0.9532	0.9809	0.9815	0.9787	0.9850	0.9878	0.9870	1.2155	1.2334	1.1952
			(0.2114)	(0.2096)	(0.2112)	(0.1370)	(0.1346)	(0.1443)	(0.4918)	(0.4922)	(0.4922)	(0.6058)	(0.6132)	(0.5972)
		2.0:2.0	0.9507	0.9509	0.9509	0.9721	0.9730	0.9701	3.8267	3.8331	3.8332	4.6457	4.7129	4.5846
			(0.2166)	(0.2162)	(0.2160)	(0.1648)	(0.1621)	(0.1702)	(4.4557)	(4.4621)	(4.4569)	(5.4160)	(5.4861)	(5.3574)
25:100	0.5:0.5	0.5:0.5	0.9807	0.9805	0.9805	0.9980	0.9983	0.9964	1.1473	1.1356	1.1419	1.6642	1.6403	1.5629
			(0.1377)	(0.1382)	(0.1382)	(0.0447)	(0.0416)	(0.0599)	(0.4957)	(0.4955)	(0.4960)	(0.6894)	(0.6797)	(0.6699)
		0.5:1.0	0.9724	0.9721	0.9727	0.9972	0.9972	0.9944	0.8671	0.8598	0.8638	1.2510	1.2349	1.1818
			(0.1638)	(0.1648)	(0.1631)	(0.0528)	(0.0528)	(0.0746)	(0.3662)	(0.3658)	(0.3660)	(0.5176)	(0.5103)	(0.5023)
		0.5:2.0	0.9658	0.9656	0.9663	0.9923	0.9923	0.9891	0.5696	0.5664	0.5680	0.7922	0.7839	0.7549
			(0.1818)	(0.1823)	(0.1806)	(0.0872)	(0.0876)	(0.1037)	(0.2272)	(0.2265)	(0.2267)	(0.3276)	(0.3229)	(0.3162)
		1.0:1.0	0.9640	0.9634	0.9635	0.9930	0.9930	0.9901	2.2570	2.2514	2.2553	3.0999	3.0705	2.9669
			(0.1863)	(0.1878)	(0.1875)	(0.0834)	(0.0834)	(0.0992)	(2.2019)	(2.2010)	(2.1948)	(2.9115)	(2.8809)	(2.8122)
		1.0:2.0	0.9609	0.9603	0.9606	0.9906	0.9897	0.9873	1.3491	1.3464	1.3485	1.8520	1.8361	1.7781
			(0.1939)	(0.1953)	(0.1946)	(0.0965)	(0.1008)	(0.1121)	(1.2328)	(1.2315)	(1.2329)	(1.6472)	(1.6301)	(1.5990)
		2.0:2.0	0.9552	0.9549	0.9555	0.9823	0.9815	0.9805	9.8520	9.8474	9.8719	13.0937	12.9932	12.6240
			(0.2069)	(0.2076)	(0.2061)	(0.1317)	(0.1349)	(0.1384)	(37.9743)	(37.9299)	(38.9848)	(50.3828)	(49.9748)	(47.9015)
	0.8:0.8	0.5:0.5	0.9679	0.9706	0.9692	0.9939	0.9945	0.9925	0.9612	0.9668	0.9632	1.3004	1.3211	1.2580
			(0.1762)	(0.1689)	(0.1728)	(0.0781)	(0.0742)	(0.0865)	(0.2960)	(0.2947)	(0.2951)	(0.3701)	(0.3718)	(0.3691)
		0.5:1.0	0.9647	0.9674	0.9655	0.9929	0.9944	0.9907	0.6682	0.6727	0.6703	0.8879	0.9034	0.8614
			(0.1845)	(0.1776)	(0.1826)	(0.0838)	(0.0746)	(0.0962)	(0.2006)	(0.1999)	(0.2003)	(0.2529)	(0.2543)	(0.2507)
		0.5:2.0	0.9601	0.9623	0.9619	0.9894	0.9911	0.9873	0.4106	0.4138	0.4127	0.5244	0.5341	0.5117
			(0.1957)	(0.1906)	(0.1914)	(0.1024)	(0.0941)	(0.1118)	(0.1216)	(0.1214)	(0.1217)	(0.1550)	(0.1562)	(0.1531)
		1.0:1.0	0.9555	0.9565	0.9554	0.9857	0.9871	0.9828	1.4915	1.4964	1.4951	1.8606	1.8917	1.8248
			(0.2061)	(0.2041)	(0.2064)	(0.1189)	(0.1127)	(0.1300)	(0.7738)	(0.7750)	(0.7759)	(0.9162)	(0.9277)	(0.9073)
		1.0:2.0	0.9554	0.9556	0.9549	0.9821	0.9833	0.9808	0.8689	0.8721	0.8713	1.0711	1.0893	1.0536
			(0.2064)	(0.2060)	(0.2075)	(0.1325)	(0.1280)	(0.1372)	(0.4425)	(0.4433)	(0.4439)	(0.5320)	(0.5389)	(0.5265)
		2.0:2.0	0.9511	0.9511	0.9509	0.9725	0.9734	0.9709	3.6075	3.6174	3.6173	4.2818	4.3503	4.2317
			(0.2157)	(0.2156)	(0.2162)	(0.1636)	(0.1609)	(0.1680)	(4.1171)	(4.1269)	(4.1252)	(4.8348)	(4.9046)	(4.7730)
50:100	0.2:0.2	0.5:0.5	0.9866	0.9860	0.9856	0.9995	0.9993	0.9989	1.4049	1.3732	1.3848	2.1540	2.0608	2.0250
			(0.1150)	(0.1175)	(0.1191)	(0.0231)	(0.0258)	(0.0326)	(0.8365)	(0.8315)	(0.8302)	(1.2790)	(1.2177)	(1.2207)
		0.5:1.0	0.9806	0.9796	0.9798	0.9978	0.9977	0.9970	1.1619	1.1386	1.1468	1.7766	1.7053	1.6790
			(0.1379)	(0.1414)	(0.1407)	(0.0469)	(0.0483)	(0.0547)	(0.6260)	(0.6216)	(0.6199)	(0.9955)	(0.9475)	(0.9471)

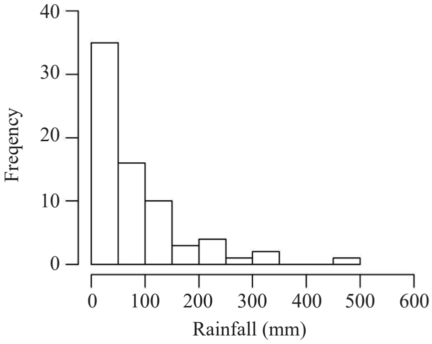
Table 2: The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the ratio of two independent coefficients of variation of the delta-lognormal distribution: unequal sample sizes (*Continued*)

$n_1 : n_2$	$\delta_1 : \delta_2$	$\sigma_1^2 : \sigma_2^2$	Coverage Probabilities (Standard Deviation)						Expected Lengths (Standard Deviation)					
			$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$	$CI_{gci.vst}$	$CI_{gci.w}$	$CI_{gci.j}$	$CI_{m.vst}$	$CI_{m.w}$	$CI_{m.j}$
		0.5:2.0	0.9661	0.9651	0.9659	0.9935	0.9928	0.9909	0.8576	0.8438	0.8486	1.2547	1.2113	1.1933
			(0.1811)	(0.1836)	(0.1814)	(0.0802)	(0.0846)	(0.0948)	(0.4124)	(0.4080)	(0.4094)	(0.6786)	(0.6458)	(0.6399)
		1.0:1.0	0.9749	0.9739	0.9745	0.9977	0.9975	0.9962	3.2880	3.2586	3.2687	5.0053	4.8082	4.7544
			(0.1563)	(0.1593)	(0.1575)	(0.0476)	(0.0503)	(0.0615)	(4.9844)	(4.9549)	(4.9770)	(7.5763)	(7.2449)	(7.2298)
		1.0:2.0	0.9674	0.9667	0.9667	0.9937	0.9936	0.9925	2.2248	2.2057	2.2107	3.4168	3.2894	3.2540
			(0.1776)	(0.1795)	(0.1793)	(0.0793)	(0.0797)	(0.0861)	(4.9992)	(4.9472)	(4.9356)	(7.3451)	(7.0216)	(7.3161)
		2.0:2.0	0.9620	0.9613	0.9618	0.9895	0.9890	0.9874	37.9473	37.7594	38.0251	58.8394	56.4330	56.1167
			(0.1912)	(0.1930)	(0.1917)	(0.1018)	(0.1043)	(0.1115)	(438.8534)	(435.7528)	(454.8939)	(679.7391)	(649.4999)	(651.3438)
50:100	0.5:0.5	0.5:0.5	0.9805	0.9807	0.9807	0.9987	0.9987	0.9981	0.7615	0.7564	0.7589	1.0990	1.0895	1.0596
			(0.1382)	(0.1377)	(0.1377)	(0.0356)	(0.0356)	(0.0432)	(0.1731)	(0.1726)	(0.1730)	(0.2455)	(0.2431)	(0.2443)
		0.5:1.0	0.9729	0.9725	0.9727	0.9975	0.9973	0.9961	0.6080	0.6051	0.6067	0.8587	0.8525	0.8321
			(0.1625)	(0.1634)	(0.1629)	(0.0503)	(0.0516)	(0.0621)	(0.1332)	(0.1327)	(0.1329)	(0.1926)	(0.1908)	(0.1904)
		0.5:2.0	0.9615	0.9612	0.9614	0.9902	0.9901	0.9877	0.4389	0.4377	0.4384	0.5777	0.5746	0.5638
			(0.1923)	(0.1931)	(0.1927)	(0.0985)	(0.0992)	(0.1101)	(0.0940)	(0.0936)	(0.0938)	(0.1371)	(0.1359)	(0.1347)
		1.0:1.0	0.9669	0.9663	0.9662	0.9942	0.9939	0.9930	1.1635	1.1610	1.1621	1.5928	1.5840	1.5542
			(0.1790)	(0.1804)	(0.1807)	(0.0759)	(0.0777)	(0.0834)	(0.4636)	(0.4633)	(0.4628)	(0.6001)	(0.5967)	(0.5919)
		1.0:2.0	0.9574	0.9568	0.9568	0.9887	0.9886	0.9873	0.7824	0.7813	0.7819	1.0335	1.0288	1.0122
			(0.2020)	(0.2033)	(0.2033)	(0.1056)	(0.1062)	(0.1118)	(0.2975)	(0.2972)	(0.2972)	(0.3940)	(0.3918)	(0.3870)
		2.0:2.0	0.9557	0.9555	0.9557	0.9851	0.9845	0.9830	2.5526	2.5513	2.5537	3.2763	3.2644	3.2183
			(0.2058)	(0.2061)	(0.2058)	(0.1213)	(0.1234)	(0.1293)	(2.1223)	(2.1212)	(2.1242)	(2.6723)	(2.6618)	(2.6256)
	0.8:0.8	0.5:0.5	0.9683	0.9697	0.9697	0.9957	0.9959	0.9947	0.6655	0.6672	0.6655	0.9124	0.9174	0.8916
			(0.1753)	(0.1715)	(0.1713)	(0.0652)	(0.0642)	(0.0724)	(0.1238)	(0.1233)	(0.1236)	(0.1598)	(0.1598)	(0.1609)
		0.5:1.0	0.9641	0.9651	0.9641	0.9929	0.9934	0.9916	0.4831	0.4848	0.4837	0.6378	0.6425	0.6262
			(0.1861)	(0.1836)	(0.1860)	(0.0838)	(0.0810)	(0.0913)	(0.0893)	(0.0891)	(0.0892)	(0.1161)	(0.1164)	(0.1160)
		0.5:2.0	0.9559	0.9563	0.9559	0.9831	0.9838	0.9818	0.3218	0.3230	0.3224	0.3972	0.4004	0.3920
			(0.2054)	(0.2044)	(0.2054)	(0.1288)	(0.1263)	(0.1337)	(0.0603)	(0.0602)	(0.0602)	(0.0781)	(0.0785)	(0.0776)
		1.0:1.0	0.9549	0.9554	0.9545	0.9869	0.9869	0.9854	0.9203	0.9215	0.9207	1.1540	1.1623	1.1396
			(0.2075)	(0.2064)	(0.2085)	(0.1136)	(0.1136)	(0.1200)	(0.2688)	(0.2687)	(0.2690)	(0.3191)	(0.3208)	(0.3172)
		1.0:2.0	0.9517	0.9522	0.9517	0.9809	0.9812	0.9795	0.5811	0.5821	0.5819	0.7020	0.7072	0.6953
			(0.2143)	(0.2134)	(0.2143)	(0.1370)	(0.1358)	(0.1418)	(0.1656)	(0.1656)	(0.1658)	(0.1989)	(0.2001)	(0.1975)
		2.0:2.0	0.9535	0.9537	0.9541	0.9751	0.9754	0.9742	1.6727	1.6744	1.6740	1.9582	1.9727	1.9453
			(0.2105)	(0.2102)	(0.2094)	(0.1557)	(0.1549)	(0.1585)	(0.8750)	(0.8757)	(0.8760)	(1.0024)	(1.0093)	(0.9976)

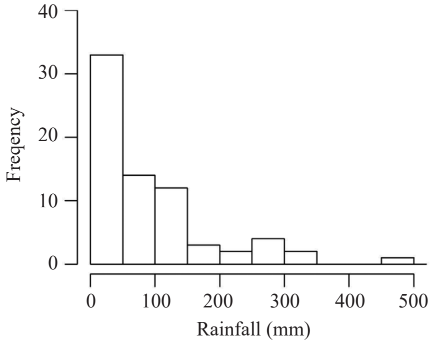
Table 3: Rainfall series (mm) from Ban Thap Tawi Water Supply and Ban Lert Sawat School stations, Si Khiu District, Nakhon Ratchasima, Thailand since April, 2010 to March, 2016

Month	Ban Thap Tawi Water Supply, Si khiu District, Nakhon Ratchasima (Station 258A1)						Ban Lert Sawat School, Si Khiu District, Nakhon Ratchasima (Station 257A1)					
	2010	2011	2012	2013	2014	2015	2010	2011	2012	2013	2014	2015
April	107.3	289.4	81.2	55.4	114.8	25.8	95.1	115.9	99.2	108.2	50.8	23.8
May	51.2	245.8	153.2	87.9	36.0	36.7	19.6	223.5	115.5	111.6	186.3	53.5
June	148.8	169.8	128.3	138.5	49.1	101.3	67.3	28.4	41.9	131.5	56.2	64.3
July	123.1	80.1	64.1	111.9	75.0	48.0	123.9	67.5	95.2	84.6	40.3	99.6
August	137.7	104.9	89.6	88.6	100.2	74.2	244.1	95.4	65.0	31.8	148.7	145.6
September	292.2	301.9	204.7	454.7	252.2	185.5	124.3	229.5	329.9	340.0	91.9	246.1
October	564.1	252.2	64.0	318.1	16.0	109.1	454.4	144.7	27.8	276.3	55.6	165.9
November	0.0	10.0	18.3	0.0	20.6	8.8	0.0	0.0	34.0	3.5	62.0	26.2
December	0.0	0.0	0.0	0.0	15.0	40.0	0.0	0.0	0.0	0.0	6.0	20.5
January	0.0	21.1	12.5	0.0	3.5	8.6	0.0	19.5	5.8	0.0	22.8	37.3
February	82.5	11.3	0.0	0.0	23.3	0.0	17.8	0.0	0.0	0.0	35.7	0.0
March	3.3	21.4	25.5	78.4	68.8	16.3	2.4	156.4	33.5	0.0	11.2	0.0

Note: This rainfall series collected from website of Lower Northeastern Region Hydrological lrrigation Center (Nakhon Ratchasima, Thailand). (http://hydro-4.com/3rainfalldata/rainmonth/rainmonth.htm)

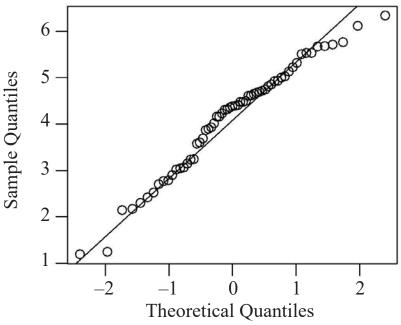


(a) Ban Thap Tawi Water Supply station

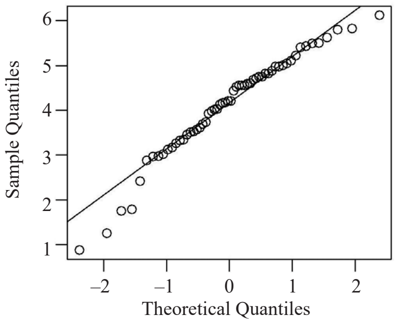


(b) Ban Lert Sawat School station

Figure 1: The density of rainfall series from Ban Thap Tawi Water Supply and Ban Lert Sawat School stations, Si Khiu District, Nakhon Ratchasima, Thailand.



(a) Ban Thap Tawi Water Supply station



(b) Ban Lert Sawat School station

Figure 2: The normal Q-Q plots of log-transformed datasets from Ban Thap Tawi Water Supply and Ban Lert Sawat School stations, Si Khiu District, Nakhon Ratchasima, Thailand.

Table 4: AIC results to check the distributions of positive values from Ban Thap Tawi Water Supply and Ban Lert Sawat School stations, Si Khiu District, Nakhon Ratchasima, Thailand

Stations	Distributions					
	Normal	Lognormal	Cauchy	Exponential	Weibull	Gamma
Ban Thap Tawi Water Supply	835.6944	830.9085	863.5236	1137.0130	851.0799	831.7608
Ban Lert Sawat School	790.7373	788.9021	825.2753	1126.2610	802.4189	789.0713

4 Discussion and Conclusions

In this study, we investigated methods for confidence interval construction consisting of VST for GCI and MOVER, the Wilson score method for GCI and MOVER, and Jeffrey's method for GCI and MOVER for the ratio of the coefficients of variation of two delta-lognormal distributions. The performance of the confidence intervals evaluated via coverage probabilities and expected lengths reveal that GCI based on VST, the Wilson score method, and Jeffrey's method were optimal as the coverage probabilities were close to the target in almost all cases and with the shortest expected lengths compared to the other methods. Indeed, overestimation occurred with the MOVER methods due to the coverage probabilities approaching 1, which is not ideal. Therefore, the GCI methods are recommended for constructing the confidence intervals for the ratio of the coefficients of variation of two delta-lognormal distributions. Moreover, the results of an empirical study coincided with those of the simulation study. Thus, we can infer that the GCI methods are appropriate for establishing confidence intervals for the ratio of the coefficients of variation when comparing two rainfall series.

Note that in cases of $n_1 : n_2 = 50{:}50, 50{:}100, \delta_1 : \delta_2$

= 0.2:0.2 and $\sigma_1^2 : \sigma_2^2$ = 2.0:2.0, the standard deviations of expected lengths were significant high. It caused of the expected number of the positive observations was small which is coincident with Fletcher [9]; moreover, the variances were large. Thus, the simulation study would not be expected to work well for such cases.

As a final remark, these results corresponded with those of Buntao and Niwitpong [21] in that GCI is the best choice for constructing confidence intervals for the ratio of the coefficients of variation of two delta-lognormal distributions. Whereas they proposed GPA based on the Wald method which worked well with large sample sizes (n_1, $n_2 \geq 100$), our approach performed well with varying sample sizes.

Table 5: The 95% confidence intervals for the ratio of coefficients of variation of rainfall series from Ban Thap Tawi Water Supply and Ban Lert Sawat School stations, Si Khiu District, Nakhon Ratchasima, Thailand

Methods	The confidence intervals for ζ		
	Lower	Upper	Length
$CI_{gci.vst}$	0.6482	1.6488	1.0006
$CI_{gci.w}$	0.6482	1.6494	1.0012
$CI_{gci.j}$	0.6492	1.6594	1.0102
$CI_{m.vst}$	0.5870	1.8080	1.2210
$CI_{m.w}$	0.5863	1.8113	1.2250
$CI_{m.j}$	0.5997	1.7757	1.1760

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