

The APTA method for the tolerance analysis of products – comparison of capability-based tolerance and inertial tolerance

Gayton N.

Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries BP 10448 F-63000 Clermont-Ferrand, France.

Beucaire P

Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries BP 10448 F-63000 Clermont-Ferrand, France

Duc E.

Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries BP 10448 F-63000 Clermont-Ferrand, France

Lemaire M.

Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries BP 10448 F-63000 Clermont-Ferrand, France

Abstract

In mass production, tolerance analysis is a very important but complex task to assess the impact of allowed geometrical deviations on the functionality of the assembled products. In the design stage, the result of tolerance analysis can be a predicted defect probability expressed in ppm (parts per million) which value is highly dependent on the probabilistic modeling of dimension deviations. Tolerance analysis meets a double problem. The first one is the geometrical description of deviations and the second one is the associated statistical model. This paper focuses on this second issue. It proposes a relevant probabilistic model of each dimension deviation since no measure is available in the design stage. Lots of authors have proposed to compute the defect probability from one particular production batch with assumptions on probabilistic laws, on mean values and on standard deviations in order to assess what we call, in this paper, a conditioned defect probability. Due to tool wear, tool settings, material variations, ... the production batches have variable probabilistic characteristics. The APTA methodology [2], proposed by the authors, aims at considering all the allowable production batches in the defect probability prediction thanks to a joint density function. The aims of this paper are to present the bases of the APTA methodology, to prove that it works for usual capability-based tolerance and inertial tolerance [3] and to compare both tolerance approaches on applications.

Keywords: *tolerance analysis, APTA methodology, inertial tolerance, capability, defect probability.*

1 Introduction

In industry, the customer's technical requirements regarding an assembled product are translated into specifications. These specifications list functional requirements to guarantee the performance of the delivered assembled product. These functional requirements are justified by using mathematical equations that can be explicit linear for a linear stack-up, explicit non-linear, or even non explicit for hyperstatic assembling or for CAD-based models. In mass production, quality requirements are necessary in order to guarantee to the customer that the delivered assembled product is robust with respect to manufacturing variability and geometrical tolerances. For each functional requirement Y , depending on part dimensions X_i , $Y = f(X_i)$, it exists a probability that the functional requirement will not be reached, that is the defect probability expressed in parts per million (ppm). In a more and more competitive world, industrial companies feel the need to tolerance analysis managing defect probability P_D in the design stage for economic and environmental reasons, reducing warranty returns and wastage in production. The calculation of such a probability would enable design tolerancing to be managed or optimized by proposing the most economic design (target value and / or tolerance of component dimensions) with respect to the allowable defect probability. This paper deals with the issue of defect probability P_D assessment in the design stage. From a scientific point of view, the calculation of P_D is not simple and concerns the assessment of a very low probability (a few ppm). To compute defect probability, assumptions on statistical models must be made, and several assumptions exist. A very interesting overview is proposed by [1]. All are based on the consideration of a particular production batch with particular values of mean values and standard deviations. However, these quantities are variable

with time due to tools wear, tools settings, material variations, ... and the new proposed APTA (Advanced Probability - based Tolerance Analysis of products) method [2] aims at taking this allowable variations with time into account in the defect probability assessment. In the industry the tolerance of part batches are specified mainly using capability requirements. Anyway, another recent possibility based on the Taguchi loss function is the inertial tolerance proposed by Pr. M. Pillet [3] for a better management of the quality of products.

This proposed paper aims at:

- presenting the basis of a new method (the APTA method) and the computation principles for explicit linear or non linear functional requirements.
- showing that it works for usual capability-based tolerance and the recent inertial tolerance;
- comparing capability-based tolerance and inertial tolerance in terms of defect probability of isostatic assembled products.

These three objectives constitute the original aspects of this paper.

After Section 2, which focuses on the conformity domain of capability-based tolerance and inertial tolerance, Section 3 gives the basis of the APTA methodology for any conformity domain and any distribution (variation) of batches characteristics. The applicability of the APTA methodology for capability-based tolerances or inertial tolerances is demonstrated for a basic one dimension problem (Section 4), for a two dimensions linear stack up (Section 5) and for a non linear functional requirement (Section 6). In all the presented APTA applications, a comparison between capability-based tolerance and inertial tolerance is proposed. Applications in sections 5 and 6 are quite simple

regarding complex problems (2D or 3D with or without gaps) presented in the literature [4,5]. The aim is not to present a complex deviation description or a complex 2D or 3D application but to deal with the consideration of variable statistical modeling in the defect probability assessment. Adaptation of the APTA methodology to deal with complex systems will be discussed in conclusion.

2 Capability-based tolerance and inertial tolerance

In both context of capability-based tolerance and inertial tolerance [3], a part dimension X_i is defined by:

- a target value T_i ;
- a tolerance t_i supposed to be shared around T_i .

A X_i production batch is defined by:

- a mean value μ_i measured on a X_i batch sample from which the mean shift δ_i can be deduced by $\delta_i = \mu_i - T_i$;
- a standard deviation noted σ_i .

2.1 Capability-based tolerance

In the context of the capability-based tolerance, two capability level requirements noted $C_{pi}^{(r)}, C_{pki}^{(r)}$ are added. Each X_i production batch must verify $C_{pi} \leq C_{pi}^{(r)}$ and $C_{pki} \leq C_{pki}^{(r)}$. C_{pi}, C_{pki} are defined with the well-known equations:

$$C_{pi} = \frac{t_i}{6\sigma_i} \quad C_{pki} = \frac{t_i/2 - |\delta_i|}{3\sigma_i}$$

In this context, the conformity domain where a production batch has conform statistical characteristics can be drawn in a standard δ, σ diagram. The conformity domain is bounded by the equations $C_{pi}(\sigma_i) = C_{pi}^{(r)}, C_{pki}(\sigma_i, \delta_i) = C_{pki}^{(r)}$. Figure 1

shows usual iso-values of C_{pi}, C_{pki} for an arbitrary value of $t_i = 2$. In this diagram, the conformity domain has a triangular shape if $C_{pki}^{(r)} = C_{pi}^{(r)}$ and is truncated at the top if $C_{pi}^{(r)} > C_{pki}^{(r)}$. As an example, the conformity domain corresponding to $C_{pki}^{(r)} = C_{pi}^{(r)} = 1.66$ is represented in grey Figure 1.

2.2 Inertial tolerance

The inertial tolerance [3] is based on the quadratic Taguchi loss function. It aims at managing the financial loss due to scatters between each measure and its target. The inertia of a manufactured batch is defined as follows:

$$I = \sqrt{\delta^2 + \sigma^2}$$

The maximum allowable inertia is noted $I^{(r)}$ and each batch must verify $I \leq I^{(r)}$ to be suitable. Figure 2 represents the conformity domain in the case of the inertial tolerance. Conformity limit are circles which equations depend on $I^{(r)}$:

$$I = I^{(r)} \rightarrow \sigma = \sqrt{I^{(r)2} - \delta^2}$$

In Figure 2, the conformity domain corresponding to $I^{(r)} = 0.2$ with $t_i = 2$ is drawn in grey.

2.3 Short comparison of both conformity domains

The maximum allowable standard deviation $\sigma_i^{(max)}$ is got for the capability-based tolerance for $C_{pi} = C_{pi}^{(r)}$ and $\sigma_i^{(max)} = t_i / 6C_{pi}^{(r)}$. For inertial tolerance, $\sigma_i^{(max)} = I^{(r)}$. In this paper, a capability-based tolerance is considered to be equivalent to an inertial tolerance if their associated maximum allowable standard deviations are the same. The following equation comes for equivalent tolerance approaches:

$$I^{(r)} = t_i / 6C_{pi}^{(r)}$$

A geometrical comparison of associated conformity domain shows that capability one is larger and allows larger mean shift. However, it exists a small zone at

the top of the circular domain that is allowed by inertial tolerance and not by capability tolerance.

The dimension out tolerance probability $P_D = \text{Prob}(X_i \notin [T_i - t_i / 2; T_i + t_i / 2])$, is defined for a Gaussian dimension by:

$$P_D = \Phi\left(-\frac{t_i / 2 + \delta_i}{\sigma_i}\right) + \Phi\left(-\frac{t_i / 2 - \delta_i}{\sigma_i}\right) \quad (1)$$

where Φ is the cumulative density function of the standard Gaussian variable. Iso-values of P_D are drawn in Figure 3 in the δ, σ diagram. These iso-values of P_D are very close to the capability-based conformity bounds. High values of P_D are got with high values of standard deviations.

3 APTA methodology bases for the defect probability prediction

For lots of bibliography references [6,7,8,9], the probability P_D that Y will be outside its bounds $P_D = \text{Prob}(Y = f(X_i) \notin [LSL_Y; USL_Y])$ is evaluated using deterministic assumptions about δ_i, σ_i . The obtained probability is only a conditioned probability, knowing the value of δ_i, σ_i ; it is called $P_{D|\delta, \sigma}$ in the following. The objective of the proposed APTA methodology is to take into account variable mean shifts and standard deviations in the evaluation of defect probability. In other words, the aim is to compute P_D rather than only computing the conditioned probability $P_{D|\delta, \sigma}$. In the following, each dimension X_i is considered to have an independent Gaussian distribution within the production batch, with a mean shift δ_i and a standard deviation σ_i . In any case, the proposed methodology could be usable and spread to non-Gaussian and/or dependent variables.

3.1 Mathematical formulation of the APTA method

The two quantities δ_i, σ_i are considered as random variables defined by a joint probability density

function called $h_{\delta, \sigma}(\delta_i, \sigma_i)$ which depends on the production device. This density function is bounded by the conformity domain. This function is equal to zero outside the conformity domain (Figure 4) because out-of-tolerance batches are considered as being excluded. It can be defined over the whole conformity domain or over a reduced domain named variability domain $V_D \subset \text{Conformity Area}$.

Let us consider the following three events:

A: The functional requirement is not satisfied (Y is outside the tolerance);

B_i : The mean shift of the X_i batch is in the range $[\delta_i; \delta_i + d\delta_i]$ and its standard deviation is in the range $[\sigma_i; \sigma_i + d\sigma_i]$, see Figure 4 for an illustration of B_i over the capability conformity domain.

B: Event B consists of the intersection of the B_i events $\bigcap_{i=1}^n B_i \in [\delta_i; \delta_i + d\delta_i] \times [\sigma_i; \sigma_i + d\sigma_i]$, that is to say that each dimension is in the specified ranges.

The probability measurement of event B_i is then:

$$\text{Prob}(B_i) = h_{\delta, \sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i$$

$$\text{and: } \text{Prob}(B) = \prod_{i=1}^n h_{\delta, \sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i$$

with an assumption of batch parameter independence for two different dimensions.

$$\text{Then: } \text{Prob}(A | B) = P_{D|\delta, \sigma}(\delta_i, \sigma_i)$$

Consequently, using Bayes' theorem:

$$\text{Prob}(A \cap B) = \text{Prob}(A | B) \text{Prob}(B)$$

$$\text{Prob}(A \cap B) = P_{D|\delta, \sigma}(\delta_i, \sigma_i) \prod_{i=1}^n h_{\delta, \sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i$$

And finally, by extension to the whole domain V_D :

$$P_D = \int_{V_D} P_{D|\delta,\sigma}(\delta_i, \sigma_i) \prod_{i=1}^n h_{\delta,\sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i \quad (2)$$

The dimension of this integral is $2n$ (n being the number of dimensions in f). P_D is the expectation of $P_{D|\delta,\sigma}(\delta_i, \sigma_i)$ weighted by the $\prod h_{\delta,\sigma}(\delta_i, \sigma_i)$ product. It is the defect probability evaluated with the APTA method considering all the possible mean shifts and standard deviations with a joint density function defined by $h_{\delta,\sigma}(\delta_i, \sigma_i)$. All the manufactured batches are assumed to be in the conformity domain and consequently out-of-tolerance batches are not considered in the formulation. Equation (2) is the basis of the APTA methodology. In addition, according to equation (2), P_D can be bounded by the upper value of $P_{D|\delta,\sigma}(\delta_i, \sigma_i)$ when (δ_i, σ_i) vary inside the variability domain $V_D : P_D \leq \max_{\delta,\sigma \in V_D} P_{D|\delta,\sigma}(\delta_i, \sigma_i)$

This upper value, called P_D^U , is easier to compute than the whole integral defined in (2) and does not require any knowledge of $h_{\delta,\sigma}(\delta_i, \sigma_i)$. To take advantage of the APTA methodology, preliminary statistical analyses must be performed in order to determine a suitable expression for $h_{\delta,\sigma}(\delta_i, \sigma_i)$. To do so, capability monitoring is necessary and knowledge must be accumulated to characterize $h_{\delta,\sigma}(\delta_i, \sigma_i)$ for each kind of process used in manufacturing. The use of this methodology requires an effort concerning production monitoring analysis. In this way, for a new product using parts manufactured with a well-known process, it is possible to use the knowledge of the old production process to validate assumptions about $h_{\delta,\sigma}(\delta_i, \sigma_i)$. Several expressions of $h_{\delta,\sigma}(\delta_i, \sigma_i)$ can be given, depending on the statistical analysis of a particular production device. For more details on the APTA method and applications in an industrial context, the reader can refer to [2].

3.2 Application to a production device with uniform δ_i and σ_i

If the standard deviations and mean shifts can be considered simultaneously random, a joint density function $h_{\delta,\sigma}$ has to be set. Considering the case where the joint (δ_i, σ_i) density function is uniform (the most severe case), then:

$$h_{\sigma,\delta}(\sigma_i, \delta_i) = \begin{cases} \frac{1}{A_V} & \text{if } \delta_i, \sigma_i \in \text{Variability domain } V_D \\ = 0 & \text{otherwise} \end{cases}$$

where A_V is the surface of the variability domain that can be the whole conformity domain or a reduced domain included within the conformity domain. Only this kind of model will be consider in the following.

3.3 Numerical computation of P_D

Equation (2) represents the mathematical expectation of $P_{D|\delta,\sigma}(\delta_i, \sigma_i)$ with respect to the joint density function of δ_i, σ_i . To reduce computation time, Equation (2) is assessed using a Monte Carlo scheme:

$$\begin{aligned} P_D &= \int_{VD} P_{D|\delta,\sigma}(\delta_i, \sigma_i) \prod_{i=1}^n h_{\delta,\sigma}(\delta_i, \sigma_i) d\delta_i d\sigma_i \\ &= E(P_{D|\delta,\sigma}) \quad \text{Expectation operator} \\ &\approx \tilde{P}_D = \frac{1}{N} \sum_{k=1}^N P_{D|\delta,\sigma}(\delta_i^{(k)}, \sigma_i^{(k)}) \end{aligned}$$

where $\delta_i^{(k)}, \sigma_i^{(k)}$ are random vectors simulated according to $h_{\delta,\sigma}$. The number N must be set according to the 95% confidence interval on P_D :

$$\tilde{P}_D - \frac{1.96\sigma_p}{\sqrt{N}} \leq P_D \leq \tilde{P}_D + \frac{1.96\sigma_p}{\sqrt{N}}$$

σ_p is the standard deviation of the \tilde{P}_D estimation defined by:

$$\sigma_p = \tilde{P}_D \sqrt{\frac{1 - \tilde{P}_D}{N \tilde{P}_D}}$$

The confidence interval size is defined as:

$$CI_{95\%} = \frac{2 \times 1.96 \sigma_p}{\sqrt{N}}$$

4 Basic APTA application on one dimension

Let us consider only one dimension noted X defined by a target value $T=10$ and a tolerance $t=2$ around the target. The aim is to compute the defect probability on X i.e. $P_D = \text{Prob}(X \notin [9;11])$ when X has capability-based tolerance or inertial tolerance. For this application, the proposed requirements on X batches are the following:

- Inertial tolerance: two cases $I^{(r)} = 0.2$ and $I^{(r)} = 0.33$;
- Capability-based tolerance: two cases $C_{pk}^{(r)} = C_p^{(r)} = 1.66$ and $C_p^{(r)} = C_{pk}^{(r)} = 1$ that are equivalent to inertial tolerance in the maximum allowable standard deviation.

In both cases, the minimum standard deviation in optimal manufacturing conditions is assumed to be $\sigma^{(\min)} = 0.03$. This is given a lower bound of the variability domain. Figure 5 shows graphically the results for both tolerance approaches. The conditioned defect probability $P_{D|\delta,\sigma}(\delta_i, \sigma_i)$ computation is performed using equation (1). Firstly, the upper bound P_D^U is computed over the variability domain in grey. For both tolerance approaches, the defect probability upper bound is got for the most important standard deviation. Consequently for equivalent tolerance approaches verifying $I^{(r)} = t_i / (6C_p^{(r)})$, the defect probability upper bound is the same in this basic application. The APTA defect probability is computed for the four variability domains considering uniform joint density $h_{\delta,\sigma}(\delta, \sigma)$. Results are presented graphically in Figure 5 in the white rectangular with its associated confidence interval between parentheses. The APTA value P_D is the expectation of conditioned defect

probability over the whole variability domain. Even if, for equivalent tolerance approach verifying $I^{(r)} = t_i / (6C_{pi}^{(r)})$, the capability variability domain is larger, the APTA defect probabilities are very close. This is explained by the fact that high conditioned defect probabilities can be met in the top of the inertial domain excluded by the capability domain. This basic application shows that capability and inertial conformity domains lead to the same number of mean out tolerance parts and to the same number of maximum conditioned out tolerance parts.

5 Linear stack up application

Let us consider a very simple mechanical assembly of two parts, 1 and 2 (Figure 6). The functional requirement of such an assembly is $Y = X_1 + X_2$. Parts 1 and 2 of the product are specified as follows:

- The target dimensions are $T_1 = 6$ for part 1 and $T_2 = 4$ for part 2.
- The tolerances on parts 1 and 2 are set to $t_1 = t_2 = 1 / (1.2\sqrt{2}) = 0.59$, corresponding to the modified root sum of squares tolerancing method [6]. A different choice could have been made.
- The capability requirements are $C_{pi}^{(r)} = 1$ and $C_{pki}^{(r)} = 1$ for each part. The probability results will be compared to an equivalent inertial tolerance $I_i^{(r)} = 0.098$ for each part. The standard deviation lower bound is arbitrary set to $\sigma^{(\min)} = 0.032$ corresponding to a maximum capability value $C_p^{(\max)} = 3$.
- The target value on Y is $T_Y = 10$ and the functional tolerance is $t_Y = 1$ i.e. $LSL_Y = 9.5, USL_Y = 10.5$.

5.1 Conditioned defect probability computation
 $P_{D|\delta,\sigma}(\delta_i, \sigma_i)$

For any linear stack up application with Gaussian random dimensions, the defect probability knowing

the mean shift and the standard deviation (δ_i, σ_i) of each variable X_i is defined as follows:

$$P_{D|\delta,\sigma}(\delta_i, \sigma_i) = \Phi\left(-\frac{\mu_Y - LSL_Y}{\sigma_Y}\right) + \Phi\left(-\frac{USL_Y - \mu_Y}{\sigma_Y}\right)$$

μ_Y, σ_Y are respectively the mean shift and the standard deviation of the resultant dimension Y function of (δ_i, σ_i) . For this particular application:

$$\begin{aligned} \mu_Y &= T_Y + \delta_1 + \delta_2 \\ \sigma_Y &= \sqrt{\sigma_1^2 + \sigma_2^2} \end{aligned}$$

5.2 Upper bound probability

Results are presented graphically in Figure 7. Upper bound results depend only on the absolute value of mean shift of each part dimensions. For the capability-based tolerance, the upper bound of defect probability P_D^U is got for the maximum allowable mean shift that is to say for the minimum value of standard deviation. For inertial tolerance, the upper value is obtained for a particular combination of mean shift and standard deviation located at the frontier of the variability domain. The capability domain upper value is higher than the inertial domain upper value. Furthermore, for capability-based tolerance, the upper value is linked to the minimum reachable standard deviation which is difficult to manage in production. The smallest is the standard deviation, the highest is the upper bound of defect probability. That is an important drawback of the capability-based tolerance regarding inertial tolerance that avoids high mean shifts and consequently high conditioned defect probabilities.

5.3 APTA results with uniform mean shifts and standard deviations

For both tolerance approaches, Figure 7 summarizes the results got with the APTA approach. The expectation of defect probability from the whole

variability domain is slightly smaller for inertial tolerance than for capability-based tolerance.

6 Non linear application – study of a one way clutch

The mechanical system studied is a one-way clutch [10] (see Figure 8). A one-way clutch transmits torque in a single direction. The clutch assembly consists of the following components: a hub, an outer ring, four rollers, and four springs. The geometry of the system is defined by three main part dimensions noted A, C, E . The characteristics of each part dimension, presented in Table 1, were adapted from [10] where only standard deviations were given. As previously, two tolerance approaches are proposed with a minimum standard deviation $\sigma_i^{(min)}$ reachable in production. The function of the one-way clutch mechanism is governed by the pressure angle noted ϕ_1 , that takes the following non linear analytic expression:

$$\phi_1 = \cos^{-1}\left(\frac{A+C}{E-C}\right)$$

For functional reasons, the pressure angle must be in the range $[6.4184; 7.6184]$ in degrees. The defect probability is defined by:

$$P_D = \text{Prob}(\phi_1 \notin [6.4184; 7.6184]).$$

6.1 Conditioned defect probability computation

$$P_{D|\delta,\sigma}(\delta_i, \sigma_i)$$

This application deals with a non-linear function f . The computation of $P_{D|\delta,\sigma}$ can always be carried out by Monte Carlo simulations, but this calculation may quickly become very time-consuming especially when repeated lots of times as in the APTA method. To reduce calculation time, the FORM approximation [11] is proposed to be used to evaluate $P_{D|\delta,\sigma}$ as other authors proposed before [9, 12]. This consists of a linearization of the limit state functions

$G_1(X_i) = f(X_i) - LSL_y$ and $G_2(X_i) = USL_y - f(X_i)$ around the most central failure point.

6.2 Upper bound probability

The results of upper bound probability over the variability domain are summarized in Figure 9 (top for capability-based tolerance and bottom for inertial tolerance). In a non linear case, the search of the maximum value is not easy since each variable has its own influence on f . For the capability-based tolerance the maximum value is located at the maximum allowable mean shift for each dimension. However, the sign of the mean shift is important because only one combination (negative for A, C and positive for E) lead to the upper value of defect probability. For the inertial tolerance, the search is more complex because the location is not known *a priori*. Anyway the maximum value $P_D^U = 336979\text{ppm}$ achieved in the inertial domain is smaller than the capability one.

6.3 APTA results with uniform mean shifts and standard deviation

The results of the APTA methods can be found graphically in Figure 9. The expectation of the defect probability is greater for the inertial tolerance than for the capability-based tolerance. Additional computations confirm that this is due to a high value of $\sigma_i^{(\min)}$ chosen in this case in comparison with the linear stack up application. Very high punctual defect probabilities are located in the low corners of the capability variability domain. A high minimum standard deviation avoids high conditional defect probabilities and consequently decreases the expectation of the defect probability.

7 Conclusions

The bases of the APTA method are reminded in this paper. For more information on the applicability in an industrial context, the reader can refer to [2] where

more significant industrial applications are proposed from capability-based tolerance of parts. This paper shows the applicability of the APTA method on linear stack up and non linear functional requirement. The second aim of this paper was to prove the applicability of the APTA methodology in the case of inertial tolerance. Furthermore, the APTA method can give results with mixed capability-based and inertial tolerance on parts of the same mechanism. The third aim was to provide a comparison of capability-based tolerance and inertial tolerance in terms of defect probability brought about both approaches. This comparison, provided from applications, is performed by a study of conditioned defect probability over the variability domain. The inertial tolerance is very interesting because it avoids very high conditioned defect probabilities banning high values of mean shifts. The inertial tolerance restricts the conformity domain size but restricts also the maximum conditional defect probability that is very important in an industrial context. However, even if the capability domain is larger than the inertial domain, expectation of defect probability provided by the APTA method can be higher for inertial tolerance due to high values of allowed defect probability at the top of the circular variability domain. From the bases of the APTA method, the perspectives are multiple and can consider three ways. The first one is to use the APTA method for the defect probability sensitivity analysis (to define critical dimensions to monitored in production) or for tolerance synthesis minimizing a cost function subjected to a given defect probability. An important issue to use the APTA method is to characterize the joint probability density function $h_{\delta, \sigma}(\delta_i, \sigma_i)$ from batch data's following process type. This constitutes the second way of improvement. The last one is to go about complex mechanisms with improved deviation description. The main issue is the computation time.

Problem treated in [5] deals with very time consuming Monte Carlo simulations. The APTA methods needs lots of probability evaluations with different statistical distributions (see paragraph 3.3). An alternative to Monte Carlo simulation has to be found to tackle tolerance analysis of complex systems with the APTA method.

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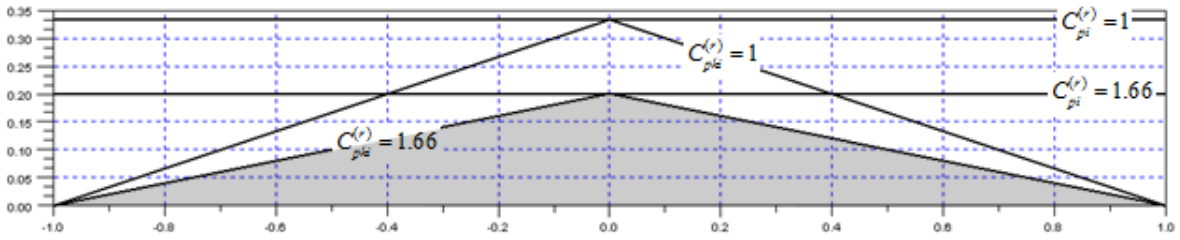


Figure 1 : Capability-based tolerance, representation of the conformity domain. In grey conformity domain with $t = 2, C_{pk}^{(r)} = C_p^{(r)} = 1.66$.

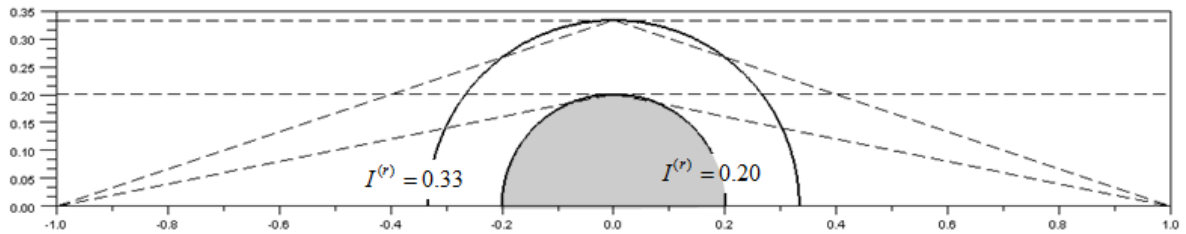


Figure 2 : Inertial tolerance, representation of the conformity domain. In grey conformity domain with $t = 2, I^{(r)} = 0.20$.

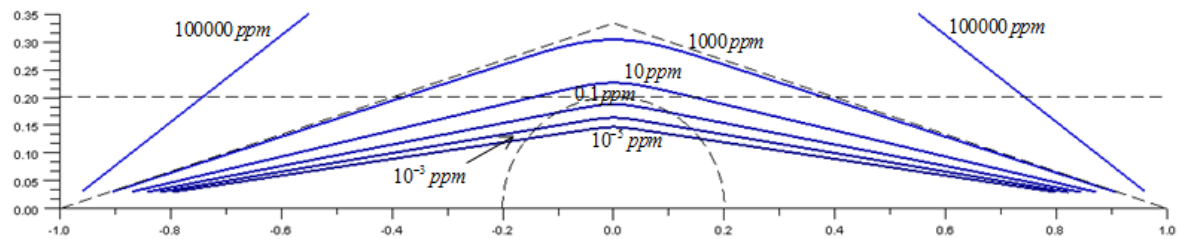


Figure 3 : comparison of capability-based tolerance and inertial tolerance, iso-defect probability $\text{Prob}(X_i \notin [LSL_i, USL_i])$ in ppm.

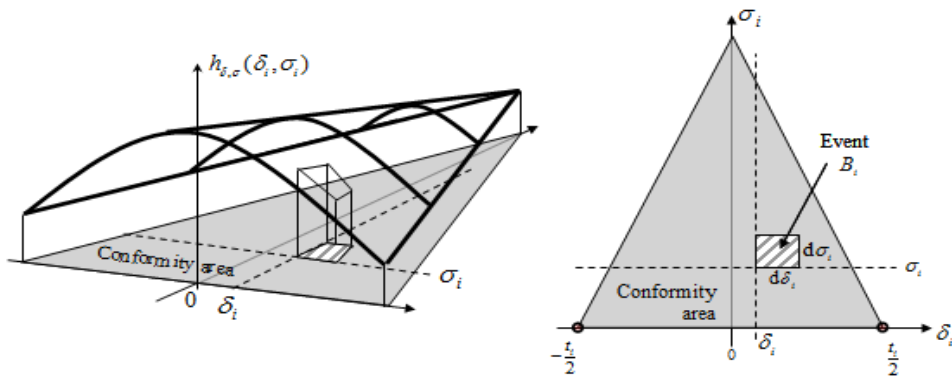


Figure 4 : Illustration on capability-based conformity domain of the joint probability density function $h_{\delta, \sigma}(\delta_i, \sigma_i)$ on the left and event B_i on the right.

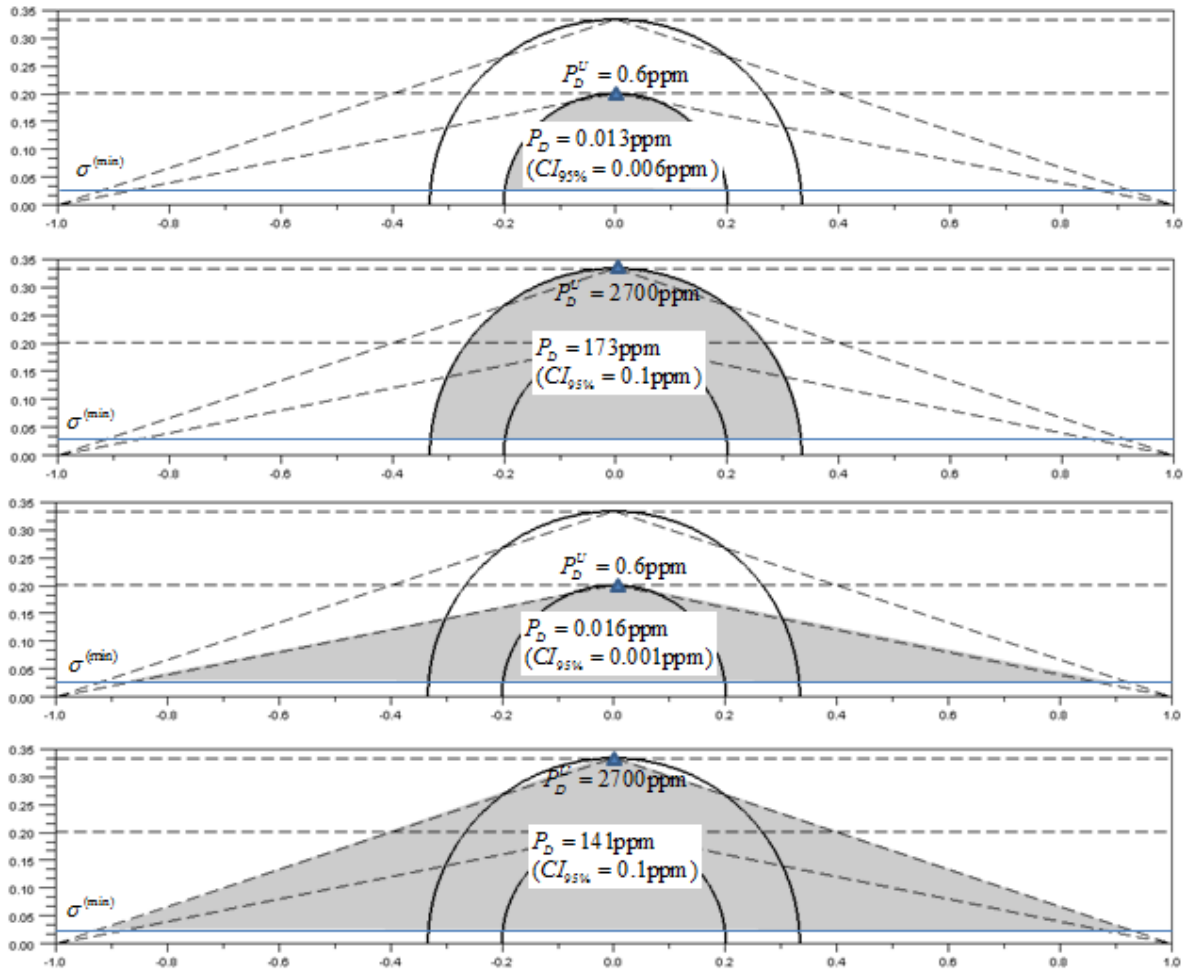


Figure 5 : APTA methodology applied to evaluate $\text{Prob}(X \notin [LSL, USL])$. Uniform distribution within the variability domain in grey bounded by $\sigma^{(\min)} = 0.03$.

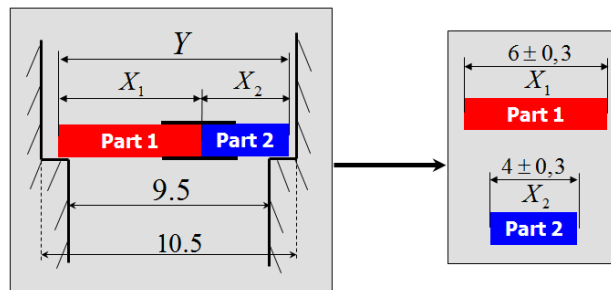


Figure 6 : Linear stack up application.

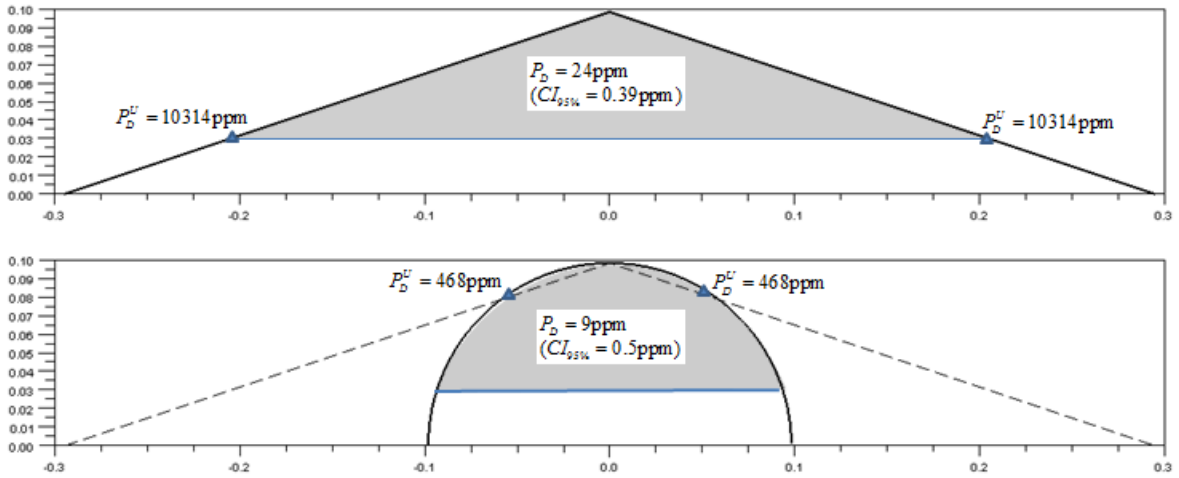


Figure 7 : Capability-based tolerance (top) vs inertial tolerance (bottom) - APTA method applied on the basic two dimensions linear stack up. Variability domain in grey.

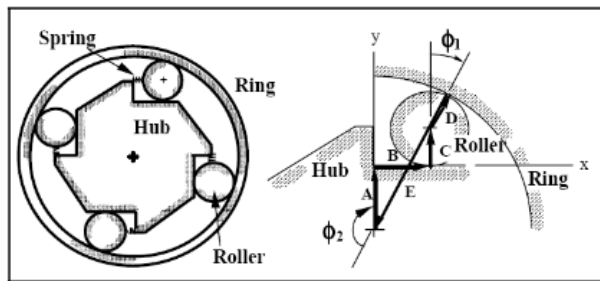


Figure 8 : Non linear application – one way clutch mechanism.

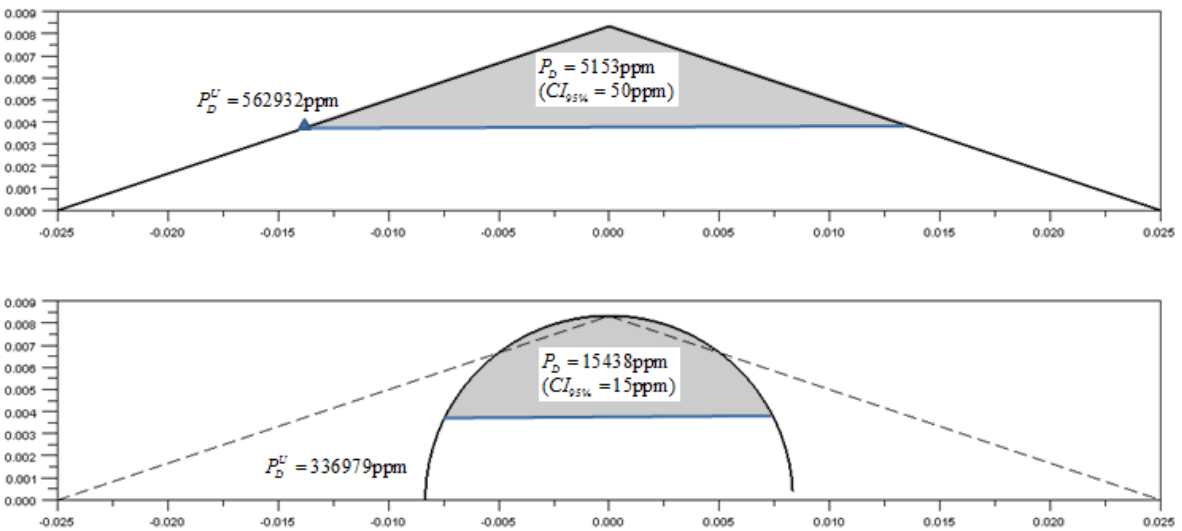


Figure 9 : Capability-based tolerance (top) vs inertial tolerance (bottom) - APTA method applied on the one way clutch. Variability domain in grey.

Table 1 : dimensions characteristics for the one way clutch.

Dim.	T_i	t_i	$C_{pi}^{(r)}$	$C_{pki}^{(r)}$	$I^{(r)}$	$\sigma^{(\min)}$
A	27.645	0.05	1	1	0.0083	0.0042
C	11.43	0.05	1	1	0.0083	0.0042
E	50.80	0.05	1	1	0.0083	0.0042