Eigenvalue Trajectory and the Space Curvature Adaptation on Improving Tactile Recognition Process

Petchartee S.
University of Applied Sciences Regensburg, Regensburg, Germany, somrak.petchartee@gmail.com

Suebsomran A.
King Mongkut's University of Technology North Bangkok, Thailand, asr@kmutnb.ac.th

Gareth M.
University of Applied Sciences Regensburg, Germany, gareth.monkman@e-technik.fh-regensburg.de

Abstract
The authors have proposed a novel surface recognition algorithm capable of determining contact surfaces types by means of tactile sensor fusion since 2008. With Quadric surface representing; the limiting procedures which relate various degenerate Cartesian coordinate systems play a crucial result in the classification of all such systems. In this paper, the renew recognition processes are described. The space transformations are technically reducing the complicated contact surface for classification refinement. The complete classification of these modifiable coordinate systems is provided by means of the corresponding space curvature. Information is obtained directly at the interface between the object and the sensing device and relates to three-dimensional. The technique called "eigenvalue trajectory analysis", is introduced after and adopted for specifying the margin of classification. The authors demonstrate mathematically approach which offers significant computational advantages such as dynamic recognition of contact deformations.

Keywords: *Contact recognition, Space Curvature, tactile sensor, eigenvalue trajectory*

1 Introduction

The technique called "eigenvalue trajectory analysis", is introduced for the contact recognitions and adopted for specifying the margin of classification [1]. The smallest component of an eigenvalue can be used to estimate and identify object shapes without using any other references, whereas classification is used as the principal indication of surface identity. The shape reflectance parameter unique to each surface may be recovered and identified. It has been shown that the reliability of the surface classification method and the accuracy of transformation are dependent of object shapes. The quadric surfaces used still have a limited number of possibilities which can be accurately represented. The five non-singular quadrics are the ellipsoid, elliptic paraboloid, hyperbolic paraboloid, hyperboloid of one sheet, and hyperboloid of two sheets. The singular quadrics include cones, cylinders, swept hyperbolas and parabolas and the degenerates: planes. To improve classification the surface properties must be modified. One option is to parameterize the Quadric surface in higher space and obtain the higher surface discrimination while preserving the eigenvalue trajectory behavior. The authors also introduced the method to improve the classification performances with multivariate regression and perturbation on singular value decomposition [2]. There exist various tactile sensors for robotic application. The basics of each tactile sensor is when it comes into contact with an object and examine what information the sensor can provide relating to the objects characteristics. The end product will be a sophisticated property map interpreting the different characteristics of target objects by fusing tactile data.

Tactile arrays can recognize surface types on contact, making it possible for a tactile system to recognize translation, rotation, and scaling an object independently. The type of contact surfaces obtained by the tactile system will be determined from the shape of the object image which can then be characterized using the mathematical properties of contact point. Figure 1 shows an example of resistive tactile sensor for one side of the robot gripper. The tactile sensors have been developed with the following specifications: One finger consists of two 16x4 cells, two 16x2 cells, and one 6x2 cells, making up the total 204 cells for the a fingers. With resistive tactile sensor, changes in electrical resistance are detected by a tactile sensor made from electrically conductive foam. The electrical resistance measured between two electrodes on the same side of the conductive foam (one tactile element) is derived from electrical conductivity through a number of simultaneously conducting paths. Resistive tactile sensor with excellent sensitive property for robot prehension can be referred to Petchartee and Monkman works [3].

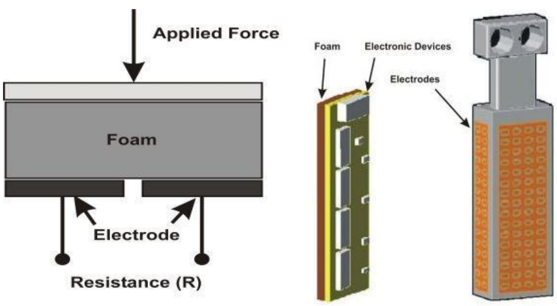

Figure 1: Sensor Components

2 Surface Fitting and Margin of Classification

2.1 Surface Fitting

The shape representation designed for this study is both rotation and translation invariant. The Quadric surface seems to be a simple, yet adequate, method for the proposed tactile sensor as the dimension of the tactile array (16x4) cannot represent a complex object surface. The basic way of creating Quadric surfaces uses least squares interpolation. Considering a general 3-D surface expressed in the contact point as $f(x,y,z)=0$, the general surface function can bapproximated locally at the contact point as the following second order polynomial equation:

$$ax^2+by^2+cz^2+2dxy+2eyz+2fxz+2gx+2hy+2jz+k=0 \quad (1)$$

Equation (1) can be rewritten in a quadratic form of matrix equation: $P^T.Q.P=0$ **,** where

$$Q=\begin{pmatrix} a & d & \underline{f} & g \\ d & b & \underline{e} & h \\ \underline{f} & \underline{e} & \underline{c} & j \\ g & h & j & k \end{pmatrix} \text{ and } \qquad P=\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

The properties of surfaces represented by Q can be translated, rotated, projected and scaled. Given a 4 x 4 transformation matrix M of the form developed, the transformed Quadric surface Q^* is:

$$Q^* = (M^{-1})^T.Q.M^{-1} \quad (2)$$

The general transformation matrices (M) are of the Denavit-Hartenberg type combining translation, rotation, scaling and projection. The least-square problem arises when the polynomial is being fit at some data points $\{(x_i,y_i)\}$, i = 1**, . . . ,** m, where m is greater than or equal to the number of unknown variables. A further generalization of the linear least-square problem is to take a linear combination of basic functions $\{f(x_1,y_1), f(x_2,y_2),..., f(x_m,y_m)\}$. Firstly, the c, e, f and j variables of Q are set to zero to get an explicit form as (3):

$$z = f(x,y) = ax^2+by^2+2dxy+2gx+2hy+k \quad (3)$$

z or $f(x,y)$ represents the tactile data of the tactile elements at the location (x,y). Then, the problem of fitting this polynomial can be initiated. In the matrix form $Ac \approx Z$, A is a square matrix, the unknown $\boldsymbol{c}$ is a column vector, and Z is also a column vector. The least-squares problem becomes: $\min\|z-Ac\|^2$. A solution of the least-squares problem is the solution $\boldsymbol{c}$ to the linear system: $A^TAc=A^Tz$, that is known as a *normal equation*. The solution of the least-squares problem is obtained by analyzing the singular value decomposition (SVD) of A.

The quadric surfaces used still have a limited number of possibilities which can be accurately represented. The five non-singular quadrics are the ellipsoid, elliptic paraboloid, hyperbolic paraboloid, hyperboloid of one sheet, and hyperboloid of two sheets. The singular quadrics include cones, cylinders, swept hyperbolas and parabolas and the degenerates: planes. To improve classification the surface properties must be modified.

2.2 Margin of Classification

The eigenvalue represents the matrix properties of the Quadric surface of object prototypes calculable from the eigenvalue trajectory of the object types. Four shapes of object have been used to test the robot's ability in recognizing object types. The robot makes contact with these objects, and the data from the tactile sensor is stored and analyzed. Later, one of the four objects is grasped again but with different magnitudes of force and in different positions and rotations. The ability to distinguish between object types is calculated. The tested objects are an oval object with two major axes of 14 mm and 11.7 mm, a cylindrical object with 6.0 mm in diameter and 20 mm in length, a cube with dimensions 10 x 15.9 x 10 mm and a ball with a diameter of 9.5 mm respectively.

This experiment applies graphical techniques to study the behavior of eigenvalues after the matrix elements change. This change normally requires numerical analysis and perturbation theory, but the technique called "eigenvalue trajectory analysis", illustrated in Figure 2, is more applicable and will be adopted. This graph shows the smallest eigenvalue of the covariance matrix of the Quadric surface property (Q) independent of translations in all two axes (along x-axis, along y-axis), of rotations around any axis (around x-axis, around y-axis, around z-axis), and of scalable values. After the trajectory of the eigenvalue is derived, it can be used to classify to the contact surface of object by matching the level of eigenvalue of surface property matrix belonging to the object prototype.

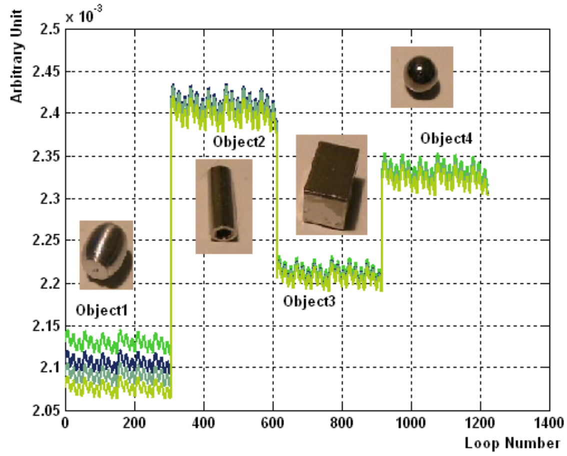

Figure 2: Eigenvalue trajectories, for different objects, derived from Quadric parameters.

Trajectory graph derived from the pseudo-codes show below;

Algorithm 1. Contact-Trajectories ($T = (N, D_{N\times4\times4})$)

1. *Initialization:*
2. ***while*** $Objnum \le N$
3. ***for*** $scale = 1; step = 0.1; scale \le MaxScaling$
4. ***for*** $tranXY = -2; step = 0.1; tranXY \le 2$
5. ***for*** $rotXYZ = 0; step = \pi/8; scale \le 2\pi$
6. $traject(i) = MinEigen(D(Objnum), scale, tranXY, rotXYZ)$
7. *end for*
8. *end for*
9. *end for*
10. *end while*
11. *DrawLineGraph(traject, MaxLoopNumber)*

The Variable i represents a LoopNumber or an index of *traject* variabe that constrains an eigenvalue at a specified transformation condition. Eigenvalues have no unit where is considered an arbitrary unit. At this point, we do not aim to prove again that the smallest eigenvalue can facilitate contact recognition, but the proof can be obtained elsewhere [1-2]. The conclusions of those proofs confirm that the smallest eigenvalue of the Quadric parameter is identical with the Quadric parameter itself. This means the smallest eigenvalue of the Quadric parameter yields identical characteristic to the Quadric shape. Consequently, it is reasonable to use the smallest eigenvalue to classify the contact surfaces. Every symmetric matrix has this property, including its covariance which is also a symmetrical matrix. General algorithms for finding eigenvectors and eigenvalues are usually iterative method, but only a few iterative methods can provide round-off errors small enough to be useful for our purposes. Powerful methods such as the QR algorithm used in the LaPACK library (Linear Algebra Software Package), have good classification ability since a precise resolution within the order of 10^{-3} is possible with very small round-off errors.

The noise level is kept below 8% it will not be statistically meaningful for, nor affect, classification. By experimenting, it is also clear that classification capability reduces if the random noise peaks are greater than 8% of the ADC's maximum value. Invalid classification was tested by increasing noise to a level higher than 8%, and consequentially, the crossing levels of eigenvalue trajectory appear. The use of the boundary alignments on the trajectory reduces the effect of noise on the eigenvalue trajectory, and such a filtration must be performed after trajectory was evaluated.

3 Previously Proposed Boundary Alignments

3.1 Multivariate regression

To improve the classification performance, the authors have experimented with the SVD mechanics analogous to the eigenvalue/eigenvector mechanics. The matrix Q can be decomposed to $Q = U\Sigma V^T$, where U and V are a set of column and row eigenvectors respectively, and Σ is a matrix of singular values (eigenvalues) of QQ^T or Q^TQ. The matrix Σ is also a quasi-diagonal matrix with its eigenvalues along the diagonal in descending order: $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0$. If we introduce an additional arbitrary matrix θ into the initial Quadric properties, then the results are $\hat{Q} = (U\Sigma V^T) + \theta$. For the different tested objects, we have differences in terms of Σ and θ. The smallest eigenvalue of the matrix Q can be aligned as $\hat{\lambda}_4^j = \lambda_4^j + e^j$, where e^j are alignment distances and j the numbering of the objects. However, V and U for the same object are not identical under the generation of a trajectory. After the eigenvalue trajectory is generated, a list of Σ_j and θ_j is produced. Then the general term of $\tilde{\theta}$ can be approximated by $\tilde{\theta} = \beta W + \varepsilon$. By substitution θ_j and Σ_j into that equation, the variable β can be obtained by using multivariate regression. In the classification process, the quadric parameter Q will be modified by $\hat{Q} = (U\Sigma V^T) + \beta\Sigma$ before the smallest eigenvalue is recalculated. The procedures above aim to align the smallest eigenvalue, in which we expect to obtain the better classification boundary. Although objects 1, 2 and 3 are uniquely classified, there exists a slight danger of misclassification in the case of objects 2 and 4 as can be seen from figure 3. According to figure 3, the thresholds of object1, object2, object3, and object4 correspond with different shapes of objects. Each object has a different eigenvalue in the eigenvalue trajectory with no particular increasing or decreasing order in terms of their levels. At this point, we are trying to enhance the level of the trajectory of object4. After doing such alighment, the unexpected results are the result of unexpectedly improved classification results as in figure 3.

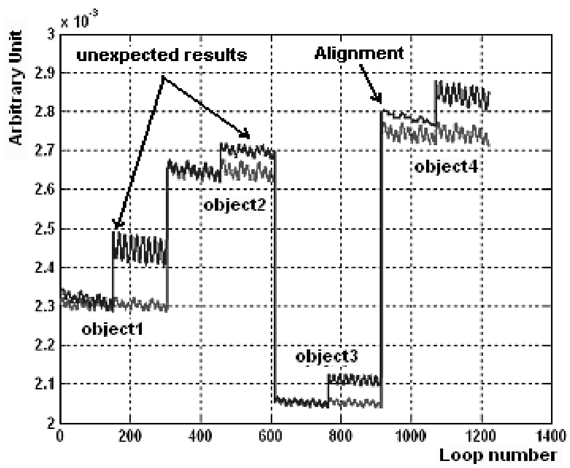

Figure 3: Boundary alignments with multivariate regression

3.2 Perturbation of SVD

The authors also introduced the method to improve the classification performances with forth-order perturbation of SVD. Based on the research work of Zhenhua (1997), the direct perturbation method of SVD with a numerical example to demonstrate the effectiveness was proposed. He advanced the second-order perturbation reanalysis. With our problems, we need forth-order perturbation for boundary alignment. Then, more proofs in Mathematics are required for further work. The SVD of a matrix Q_0^j is known as $Q_0^j = U_0^j \Sigma_0^j V_0^j$, where $\Sigma_0^j = \begin{bmatrix} \Sigma_{0r}^j & 0 \\ 0 & 0 \end{bmatrix}$ and $\Sigma_{0r}^j = diag(\lambda_{01}^j, \lambda_{02}^j, \lambda_{03}^j, \lambda_{04}^j)$. A further assumption is that another matrix Q_T^j may be derived form Q_0^j by a perturbational modification; $Q_T^j = Q_0^j + Q_p$, where Q_p is the perturbation matrix which is expected to be the common unknown alignment matrix. We concentrate our attention on forcing only the smallest eigenvalues of $Q_T^1, Q_T^2, Q_T^3, Q_T^4$ to be changed by e^1, e^2, e^3, e^4 respectively. By using the perturbational analysis method proposed by Zhenhua, we can reanalyse and introduce the general terms for the eigenvalue of Q_T^j as:

$$\begin{aligned} \hat{\lambda}_i^j &= u_{0i}^T Q_p v_{3i} + v_{0i}^T Q_p u_{3i} + (u_{0i}^T Q_p v_{0i})(u_{1i}^T u_{2i} + v_{1i}^T v_{2i}) \\ &+ (u_{0i}^T Q_p v_{1i} + v_{0i}^T Q_p u_{1i})(u_{1i}^T u_{1i} + v_{1i}^T v_{1i})/2 \end{aligned} \quad (4)$$

The terms $u_{0i}, u_{1i}, u_{2i}, u_{3i}, v_{0i}, v_{1i}, v_{2i}, v_{3i}$ could also be obtained using the basic idea of Zhenhua and our derivation in [4]. The calculation of $u_{0i}, u_{1i}, u_{2i}, u_{3i}, v_{0i}, v_{1i}, v_{2i}, v_{3i}$ are based on the contents of Q^1, Q^2, Q^3, Q^4. The subscript i is set to four for the smallest eigenvalue and to 1 for the largest. The new aligned smallest eigenvalues $\lambda_4^1, \lambda_4^2, \lambda_4^3, \lambda_4^4$ are substituted by alignment values from the previously given expression $\hat{\lambda}_4^j = \lambda_4^j + e^j$.

$$\left.\begin{aligned} 2u_{0i}^T u_{1i} &= 0 \\ 2v_{0i}^T v_{1i} &= 0 \end{aligned}\right\} \rightarrow \textit{first–order perturbation}$$

$$\left.\begin{aligned} 2u_{0i}^T u_{2i} + u_{1i}^T u_{1i} &= 0 \\ 2v_{0i}^T v_{2i} + v_{1i}^T v_{1i} &= 0 \end{aligned}\right\} \rightarrow \textit{second–order perturbation} \quad (5)$$

$$\left.\begin{aligned} 2u_{0i}^T u_{3i} + 2u_{1i}^T u_{2i} &= 0 \\ 2v_{0i}^T v_{3i} + 2v_{1i}^T v_{2i} &= 0 \end{aligned}\right\} \rightarrow \textit{third–order perturbation}$$

$$\left.\begin{aligned} 2u_{0i}^T u_{4i} + 2u_{1i}^T u_{3i} + 2u_{2i}^T u_{2i} &= 0 \\ 2v_{0i}^T v_{4i} + 2v_{1i}^T v_{3i} + 2v_{2i}^T v_{2i} &= 0 \end{aligned}\right\} \rightarrow \textit{forth–order perturbation}$$

Finally, the problem become a linear system of the form $Ax = b$ with four equations and sixteen unknown variables (the elements of Q_p). However, according to the properties of a symmetric matrix, the number of unknown variables can be reduced to ten. The 4 by 10 A matrix contains alignment coefficients. The unknown, x is a 10 by 1 column vector taken from Q_p. b is a 4 by 1 column vector of aligned eigenvalues. The solution to this problem can be solved by simple inversion: $x = A^{-1}b$. This is an underdetermined linear system which involves more unknowns than equations. The solution to such underdetermined systems is not unique. The system is underdetermined and A is not invertible, but the pseudoinverse, $A^+ = A^T(AA^T)^{-1}$, can be used to obtain the least-squares solution of $x = A^+b$. Figure 4 presents the final results of the changing of smallest eigenvalues using perturbation of SVD. Using this technique, we still face the limits of SVD perturbation bounds [5] and the effect of singular values to the singular subspace when applied to large alignments.

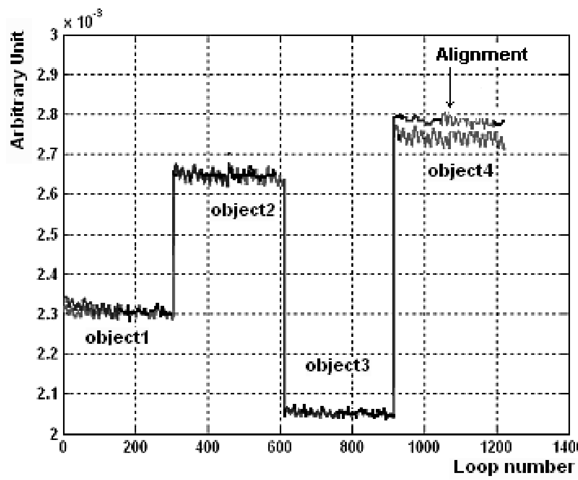

Figure 4: Boundary alignments with perturbation of SVD

4 The Space Curvature Techniques

One option is to parameterize the Quadric surface in higher space and obtain the higher surface discrimination while preserving the eigenvalue trajectory behavior. There remains the possibility that their discrimination of object shape is based on differences in surface curvature at the contact point.

A hypersurface is an n-dimensional manifold embedded in an n+1 dimensional space.

For our case, consider a two-dimensional surface embedded in three-dimensional space, where the surface consists of the locus of points satisfying an equation of the form f(x,y,z) =0. For hypersurfaces of higher dimension we can proceed in exactly the same way.

It is called a quadric surface if the surface is the equation of second degree in three-dimensional Cartesian Coordinates. However, quadric surfaces are multi-dimensional spaces defined by coordinates $\{x_0 \ \ x_1 \ \ . \ \ . \ \ x_D\}$, where the general quadric is defined by the algebraic equation,

$\sum_{i,j=0}^{\varphi} Q_{i,j} x_i x_j + \sum_{i=0}^{\varphi} P_i x_i + k = 0$, in which Q is a (φ +1)-dimensional matrix, P is a (φ +1)-dimensional vector and k a constant. For example, if a Quadric parameter is projected into a higher coordinate system, then the parameters Q' and P' will be:

$$Q' = \begin{bmatrix} a & d & f & g & w_1 \\ d & b & e & h & w_2 \\ f & e & c & j & w_3 \\ g & h & j & k & w_4 \\ w_1 & w_2 & w_3 & w_4 & 1 \end{bmatrix}, \ P' = \begin{pmatrix} x \\ y \\ z \\ w \\ 1 \end{pmatrix}$$

The curvature of an n-1 dimensional embedded in n-dimensional space as defined by the function $f(x, y, z, w,...) = 0$ is given by

$$\kappa = \frac{\Psi(A, \hat{B})}{trace(A)^{(n+1)/2}} \qquad (6),$$

where A and B are off-diagonal entries and diagonal entries of the matrix [6]. We define $\Psi(A, \hat{B}) = \sum_{i,j} (-1)^{(i+j)} A_{i,j} b_{ij}$ where b_{ij} denote the minor of $B_{i,j}$. For this case, at any point on a three-dimensional hypersurface we can construct orthogonal coordinates w, x, y, z such that the xyz hyperplane is tangent to the surface and the w axis is normal to the surface at that point.

The surface curvatures are going to decrease on the higher dimensional space by a function of the diagonal entries. For example, the curvature of an (n-1)-dimensional sphere of radius r embedded in n-dimensional space will be reduced by a factor of $1/r^{(n-1)}$.

We do not aim to prove that how other shapes decreased in surface curvature in n-dimensional space. The conclusions still confirm that the smallest eigenvalue of the Quadric parameter is identical with the Quadric parameter itself. In conclusion, Q' is still the symmetrical matrix and the smallest eigenvalue used for classification will be λ_5. w_1, w_2, w_3 and w_4 are the lists of parameters which must be found in order to project onto a higher dimensional space.

To decrease the surface curvature, the procedures above aim to align the smallest eigenvalue, In which we expect to obtain the better classification boundary.

5 Conclusions

This paper explains the authors' instruction for aligning eigenvalue trajectories. To reduce the complicated contact surface for classification and expand the windows of margin in classify process.

Reference

[1] S. Petchartee and G. Monkman, 2007. Contact Identification using Tactile Arrays. In Proceedings of the 7th IEEE International Symposium on Signal Processing and Information Technology (ISSPIT 2007), Egypt, pp. 1105-1110.

[2] S. Petchartee, G. Monkman and Anan S., 2008. 3D-Shape Recognition based Tactile Sensor. In the Book Series "Lecture Notes in Electrical Engineering", the issue titled "Smart Sensors and Sensing Technology", Springer-Verlag Berlin Heidelberg, part 9 chapter 4, pp. 317-331.

[3] S. Petchartee and G. Monkman. 2008. Optimisation of Prehension Force through Tactile Sensing. *Industrial Robot Journal*, Emerald Group Publishing Limited, Vol.35, Issue 4.

[4] Lü Zhenhua, 1997. Direct perturbation method for reanalysis of matrix singular value decomposition, Applied Mathematics and Mechanics, Vol. 18, No.5.

[5] Michael Stewart, 2006. Perturbation of the SVD in the presence of small singular values, ScienceDirect, Volume 419, Issue 1, Pages 53-77.

[6] Martin R. Bridson and Andre Haefliger, Metric Spaces of Non-Positive Curvature, Springer-Verlag Berlin Heidelberg NewYork, ISBN 3-540-64324-9