

Research Article

On Designing New Mixed Moving Average – Extended EWMA Control Chart Based on Sign Statistic

Khanittha Talordphop

Department of Mathematics and Statistics, Faculty of Science and Agricultural Technology, Rajamangala University of Technology Lanna Phitsanulok, Phitsanulok, Thailand

Saowanit Sukparungsee*

Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand

* Corresponding author. E-mail: saowanit.s@sci.kmutnb.ac.th

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Abstract

The limitations of normalcy assumptions are accommodated by a nonparametric control chart, which is user-friendly and robust. This study introduced a moving average control chart integrated with an extended exponentially weighted moving average control chart utilizing sign statistics, namely MA-EEWMA Sign. We analyzed the study for assessing the efficacy of a monitoring strategy using the average run lengths through Monte Carlo simulation. Performance comparison index (PCI), extra quadratic loss (EQL), especially overall performance are still used to evaluate the usefulness of control charts. Overall, the findings reveal that the provided chart remains the best control chart for finding moderate to minor shifts from normally distributed to skewed distribution. The effectiveness was evaluated using the following charts: moving average, exponentially weighted moving average, extended exponentially weighted moving average, and a hybrid of the latter two. The research findings were confirmed when the suggested control chart was adjusted for the actual dataset.

Keywords: Average run length, Control chart, Extended exponentially weighted moving average, Moving average, Sign test

1 Introduction

The goal of quality improvement is to make processes and products less unpredictable. Any part of an enterprise or organization can benefit from these techniques, including production, process creation, design of equipment, accounting and finance, advertising, product distribution and logistics, customer service, and maintenance [1]. When it concerns statistical process control (SPC), a control chart is a significant instrument.

This chart displays the averages of quality characteristic measurements in process trials compared to time. The normal distribution is the most often used distributional assumption for the generation of classic control charts, also known as parametric

control charts. When it comes to identifying big shifts, Shewhart's [2] first control chart is the way to go. The next generation of control charts, like EWMA, named the exponentially weighted moving average [3], the cumulative sum or CUSUM [4], the double exponentially weighted moving average (DEWMA) was introduced by Shamma & Shamma [5], the moving average chart (MA) [6], will allow for the rapid detection of shifts in development. Recently, a control chart known as the MEWMA, or modified exponentially weighted moving average [7]; and the EEWMA, or extended exponentially weighted moving average [8], control chart was presented later.

The control chart described by Alevizakos *et al.*, [9] is called a TEWMA chart, which stands for triple exponentially weighted moving average. The

homogeneously weighted moving average (HWMA) control chart was advocated by Abbas [10]. Alevizakos *et al.*, [11] created an Exp-EWMA control chart, which is an exponentially weighted moving average chart. Javed *et al.*, [12] led to the creation of the NEEWMA chart, which stands for new extended exponentially weighted moving average.

In the past decade, examiners have altered the fundamental structure of conventional charts to enhance their efficacy. Combination charts successfully track variations in process parameters. In this regard, the effectiveness of the hybrid DMA-EWMA control chart was compared to that of EWMA and MA charts, with an emphasis on their run length features, by Aslam *et al.*, [13]. Later, the EWMA-CUSUM and CUSUM-EWMA charts, created by Osei-Aning *et al.*, [14], is able to follow the correlated process better than traditional control charts, especially when it comes to identifying small to moderate changes. Likewise, based on statistics including covariance terms, Raza *et al.*, [15] created an EWMA mixed MA control chart, and the results demonstrate that it outperforms its predecessors. Furthermore, in order to evaluate performance according to average run length, Naveed *et al.*, [16] created the EEWMA-MA control chart that makes use of auxiliary data. Recently, to keep focus on the average, Talordphop *et al.*, [17] built a mixed MA-HWMA. The study shows that when compared to other methods, the suggested chart is the best at detecting changes in a procedure location parameter.

A lot of real-world data sets either do not follow a normal distribution or do not know what one is. A few of its numerous advantages include the fact that you can avoid guessing at the variance when you create charts with the location parameter, being more robust and resistant to outliers, and avoiding the assumption of a particular parametric distribution on the method at hand. Nonparametric control charts are a powerful and practical solution for resolving these constraints [18]. For the purpose of tracking such skewed processes, numerous researchers have suggested nonparametric charts like the Sign statistic. For example, a control chart that was created by Yang *et al.*, [19] using EWMA-Sign. Using the sign statistic as its foundation, Yang and Cheng developed a unique nonparametric CUSUM mean chart [20]. It demonstrated superior detection capabilities for minor changes. To enhance the process's capacity to identify minute changes, Lu [21] devised the GWMA, or

generally weighted moving average Sign control chart. In cases where process data is insufficient, these results show that the nonparametric GWMA sign chart is superior to the standard GWMA graph constructed using a less out-of-control ARL. In order to assess efficiency, Incorporating the sign statistic and utilizing the average run length, Aslam *et al.*, [22] presented a modified exponentially weighted moving average control chart (MEWMA-Sign). The results showed that when it came to identifying changes, the MEWMA-Sign chart outperformed the EWMA-Sign chart. The EEWMA sign chart, recently developed by Talordphop *et al.*, [23], has the efficiency to detect shifts quickly.

However, the update procedure isn't complete without effective monitoring. One comprehensive measure for assessing the suggested control chart's effectiveness is the average run length (ARL). Control chart efficacy is still evaluated using the performance comparison index (PCI) and extra quadratic loss (EQL) in addition to overall performance.

To improve the ability to identify tiny changes in the method mean parameter, the MA-EEWMA constructed using Sign statistics was suggested, which takes use of hybrid control charts as well as its simplicity in unusual process conditions. We evaluated the control charts' performance by obtaining the run length properties through the Monte Carlo simulation. In the end, we show how the suggested chart works in practice by comparing it to existing control charts and applying it to a scenario from reality.

2 Materials and Methods

Suppose that X designate a random variable of a process characterized by a normal distribution with mean μ and variance σ^2 . Let X_{jk} , $j = 1, 2, \dots, m$ and $k = 1, 2, \dots, n$, reflect the sequence of independent, identically distributed (iid.) procedure observations. The layout of the control chart is fundamentally as follows.

2.1 MA control chart

The moving average control chart, which was developed by Khoo [4], is shown with the statistic in Equation (1):

$$MA_k = \begin{cases} \frac{X_k + X_{k-1} + X_{k-2} + \dots}{k}, & k < w \\ \frac{X_k + X_{k-1} + \dots + X_{k-w+1}}{w}, & k \geq w \end{cases} \quad (1)$$

where (w) is the period of the moving average. The control boundaries (lower control limit and upper control limit), determined by coefficient B_1 , offer the MA chart's control limitations, as shown within Equation (2):

$$LCL / UCL = \begin{cases} \mu - B_1 \frac{\sigma}{\sqrt{w}}, & k < w \\ \mu + B_1 \frac{\sigma}{\sqrt{k}}, & k \geq w \end{cases} \quad (2)$$

where B_1 represents the coefficient of control limits used for obtaining ARL_0 .

2.2 EEWMA control chart

The EEWMA control chart was derived from the EWMA [3] control chart developed by Naveed *et al.*, [7]. The EWMA statistic is outlined in Equation (3) below:

$$E_k = \omega X_k + (1 - \omega)E_{k-1}. \quad (3)$$

Subsequently, with a smoothing parameter ω ($0 < \omega \leq 1$), the starting point of E_0 is often assigned to be equivalent to the mean μ .

The control boundaries of the EWMA chart can be seen in Equation (4), which establishes the initial steady limit B_2 :

$$LCL / UCL = \mu \mp B_2 \sigma \sqrt{\frac{\omega}{2 - \omega}} \quad (4)$$

where B_2 represents the coefficient of control limits used for obtaining ARL_0 .

The EEWMA statistic is outlined in Equation (5) here:

$$EE_k = \omega_1 X_k - \omega_2 X_{k-1} + (1 - \omega_1 + \omega_2)EE_{k-1} \quad (5)$$

which $0 < \omega_1 \leq 1$ and $0 \leq \omega_2 < \omega_1$ are smoothing

parameters with values between 0 and 1, denoted as ω_1 and ω_2 , respectively. We use the values of EE_0 and X_0 as our goal mean.

The control boundaries of the EEWMA chart can be seen in Equation (6), which establishes the initial steady limit B_3 :

$$LCL / UCL = \mu \mp B_3 \sigma \sqrt{\frac{\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2(1 - \omega_1 + \omega_2)}{2(\omega_1 - \omega_2) - (\omega_1 - \omega_2)^2}} \quad (6)$$

where B_3 represents the coefficient of control limits used for obtaining ARL_0 .

2.3 MA - EEWMA control chart

After the EEWMA and MA control charts were combined, the MA-EEWMA control chart formed. The MA-EEWMA control design statistic is articulated by the following formula in Equation (7):

$$(MA - EE)_k = \begin{cases} \frac{EE_k + EE_{k-1} + EE_{k-2} + \dots}{k}, & k < w \\ \frac{EE_k + EE_{k-1} + \dots + EE_{k-w+1}}{w}, & k \geq w \end{cases} \quad (7)$$

The control boundary establishes the control limits for the MA-EEWMA chart using coefficients B_4 , illustrated in the following Equation (8):

$$LCL / UCL = \begin{cases} \mu - B_4 \sqrt{\frac{\sigma^2}{k} \cdot \frac{\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2(1 - \omega_1 + \omega_2)}{2(\omega_1 - \omega_2) - (\omega_1 - \omega_2)^2}}, & k < w \\ \mu + B_4 \sqrt{\frac{\sigma^2}{w} \cdot \frac{\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2(1 - \omega_1 + \omega_2)}{2(\omega_1 - \omega_2) - (\omega_1 - \omega_2)^2}}, & k \geq w \end{cases} \quad (8)$$

where B_4 represents the coefficient of control limits used for obtaining ARL_0 .

2.4 MA - EEWMA sign control chart

Employing sign statistics, the MA-EEWMA Sign control chart emerged through combining the EEWMA and MA control charts. Assuming that T is the reported desired value to be tracked, the discrepancy between the two sets of data would indicate.

$$Y_{jk} = X_{jk} - T.$$

The indicator variable I_{jk} can be elaborated as Equation (9):

$$I_{jk} = \begin{cases} 1, & Y_{jk} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

A potential approach to define the sign statistic S_t is using Equation (10):

$$S_k = \sum_{j=1}^n I_{jk}. \quad (10)$$

The sum of all observations in the control condition that follow the binomial distribution given a parameter $(n, p = 0.5)$ is represented by the Sign statistic. Process ratio $p = P(Y > 0)$ and control process $p = P(Y \leq T) = P(Y > T) = 0.5$ are both represented by the quantities. When $q \neq 0.5$ is included, the process becomes extremely unpredictable.

The MA-EEWMA Sign statistic is outlined in Equation (11) here:

$$(MA-EE)_{S_k} = \begin{cases} \frac{S_k + S_{k-1} + S_{k-2} + \dots}{k}, & k < w \\ \frac{S_k + S_{k-1} + \dots + S_{k-w+1}}{w}, & k \geq w \end{cases}. \quad (11)$$

The mean and variance of MA-EEWMA Sign are presented in Equations (12) and (13).

$$E(MA-EE)_{S_t} = np \quad (12)$$

and

$$V(MA-EE)_{S_t} = \begin{cases} \frac{npq}{k} \left[\frac{\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2(1-\omega_1+\omega_2)}{2(\omega_1-\omega_2) - (\omega_1-\omega_2)^2} \right], & k < w \\ \frac{npq}{w} \left[\frac{\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2(1-\omega_1+\omega_2)}{2(\omega_1-\omega_2) - (\omega_1-\omega_2)^2} \right], & k \geq w \end{cases} \quad (13)$$

Equation (14) below lays out the control constraints of the MA-EEWMA Sign chart, which establishes the initial steady limit B_5 :

$$LCL / UCL = np \mp B_5 \sqrt{\text{Var}(MA-EE)_{S_t}} \quad (14)$$

where B_5 represents the coefficient of control limits used for obtaining ARL_0 .

2.5 Performance comparison measurers

Change detection tests are commonly employed in the manufacturing sector. The average run length (ARL) functions as a method for evaluating process alterations. There are the mean points of data through the signal cease [24]–[26]. While the operation is in control, it is designated as ARL_0 ; conversely, when the operation is out of control, this is designated as ARL_1 .

The forecast is essential to identify alterations in the process as promptly as possible, indicating that ARL_1 must be enhanced to ensure the control chart's efficacy.

The ARL is described in Equation (15) below:

$$ARL = \frac{\sum_{k=1}^N RL_k}{N}. \quad (15)$$

Additionally, the extra quadratic loss (EQL) may measure the overall outcome across the method shift region $(\delta_{min}, \delta_{max})$ as well as has been accepted as a possible factor by many authors [27]–[29]. The chart with the smallest EQL score is considered particularly effective.

The EQL equation is expressed in Equation (16) that follow:

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta) d\delta \quad (16)$$

where δ_{min} and δ_{max} denote the minimum and maximum shift assessed during the process, respectively. δ denotes the magnitude of the shift that occurs in the procedure, whereas $ARL(\delta)$ denotes the ARL that appears in a chart corresponding to the specified shift.

The method for calculating EQL is as follows: 1) Define the shift range (the evaluated lowest and largest shift across the process range from 0 to 2). 2) Calculate the control chart's ARL for each shift size in the range. 3) Find the square of each shift size. 4) Combine every ARL with its matching squared shift size (v). Calculate the EQL value.

Moreover, the performance comparison index (PCI) is the ratio comparing the EQL of a chart to the EQL of the optimal chart under comparable conditions.

This can be expressed in Equation (17) as

$$PCI = \frac{EQL}{EQL_{\text{benchmark}}}. \quad (17)$$

Next, we ran Monte Carlo simulations on each control chart to determine its ARL properties. Initially, select a subset of the population at random from a predetermined distribution. Subsequently, calculate the suggested charting statistic and evaluate "B" at the $ARL_0 = 370$ value. The control boundary is thereafter computed, and the statistical values are executed. Ultimately, perform 50,000 iterations (N) to calculate the ARL.

3 Results and Discussion

3.1 Simulation results

The present section displays the proposed chart analysis derived from the aforementioned performance metrics, utilizing a sample size of 5,000 (n) and 50,000 iterations (N) in Monte Carlo simulations while maintaining an ARL_0 of 370. The simulation run length properties associated with all charts are obtained by simulations with a smoothing parameter of 0.1 and particular shifts ranging from 0 to 2 for the normal(0,1), Laplace(0,1), Exponential(1), and gamma(4,1) distributions.

Finally, we assess the efficacy of the suggested (MA-EEWMA Sign) chart in comparison to current charts, with the chart exhibiting the smallest ARL_1 deemed the most efficient. The highlighted numbers signify that the chart performed more effectively in reducing ARL_1 .

Based on the simulation results, the advised chart has significantly higher control limit characteristics than their other control charts for every distribution.

The results illustrated in Table 1, under the assumption of normal distribution, indicate that the ARL characteristic shows the proposed chart (MA-

EEWMA Sign) has slightly enhanced detectability for small to moderate shifts (0.05 to 1.00) in comparison to the EWMA, EEWMA, and MA-EEWMA charts, while the MA control chart is superior in identifying large shifts (1.50 to 2.00).

The results from the investigation of the Laplace distribution presented in Table 2 imply that the claimed chart leads its competitors for shifts between 0.05 and 1.50, although the MA chart wins for a shift of 2.00. In comparison to EWMA, EEWMA, and MA-EEWMA, the provided chart exhibits greater stability throughout all range shifts.

Furthermore, we reveal the control chart's outcomes through skewed distributions. Table 3 clearly shows that the distribution is exponential. The offered chart has somewhat superior recognition ability for changes ranging from 0.05 to 0.75, other than the MA chart dominates in shifts from 1.00 to 2.00. The suggested chart identifies movements ranging from 0.05 to 2.00 more rapidly than EWMA, EEWMA, and MA-EEWMA charts.

Our proposed graph has the lowest ARL_1 for shifts between 0.05 and 0.75 according to the gamma distribution results in Table 4, while the EEWMA chart has the best ARL_1 for shifts between 1.00 and 2.00. Within the range of shifts from 0.05 to 1.00, the suggested chart demonstrates superior performance compared to MA, EWMA, and MA-EEWMA charts.

Unlike the integrated parametric chart (MA-EEWMA), the recommended graphic can spot changes in any distribution very quickly. The suggested chart outperforms both the individual control chart and the combined parametric control chart when it comes to detecting small to moderate process changes. Moreover, the comprehensive performance metrics shown in Table 5 elucidate this conclusion. The total performance metrics from EQL and PCL values demonstrate that the suggested chart greatly surpassed all shifts across every distribution.

In the comparative analysis, we evaluate the ARL performance of the proposed chart against that of MEWMA-Sign [30], the mixed EWMA-MA sign control chart [31], an EWMA sign control chart [32], a group runs control chart utilizing sign statistics [33], and a modified EWMA sign control chart employing repetitive sampling [34]. Also, similar to the way a control chart based on a sign statistic can notice a shift in the mean process rapidly, the simulation results showed that this phenomenon is also possible.



3.2 Application

The real-life dataset comprises the count of nonconformities identified in 26 consecutive samples

of 100 printed circuit boards [1], as detailed in this section. Analyzing nonconformities frequently leads to important understandings about the root causes.

Table 1: The examination of ARL_1 results for control charts depending on the Normal distribution.

Shift	MA $B_1 = 2.88$	EWMA $B_2 = 2.89$	EEWMA $B_3 = 2.7$	MA-EEWMA $B_4 = 2.212$	MA-EEWMA Sign $B_5 = 18.346$
0	370.94	370.11	370.95	370.71	370.35
0.05	348.61	328.94	321.40	320.14	223.74
0.1	307.91	252.96	233.16	229.17	140.83
0.25	161.76	120.87	79.72	78.56	52.23
0.5	50.99	27.37	26.23	26.22	20.34
0.75	20.55	16.23	13.99	14.33	11.77
1	10.06	8.75	9.18	9.84	8.36
1.5	3.76	4.85	5.18	6.14	5.22
2	1.98	3.40	3.43	4.34	3.47

Note: The ARL_1 with the smallest values appear in bold.

Table 2: The examination of ARL_1 results for control charts depending on the Laplace distribution.

Shift	MA $B_1 = 4.40$	EWMA $B_2 = 4.70$	EEWMA $B_3 = 4.11$	MA-EEWMA $B_4 = 2.221$	MA-EEWMA Sign $B_5 = 19.39$
0	370.39	370.72	370.61	370.59	370.42
0.05	367.15	355.17	352.26	349.35	260.74
0.1	354.53	315.77	304.66	293.18	190.51
0.25	278.39	200.83	160.02	139.37	84.93
0.5	147.51	59.34	58.21	48.55	34.79
0.75	74.78	30.66	28.92	25.17	19.43
1	38.72	18.07	18.04	16.21	12.96
1.5	12.56	9.62	9.64	9.41	7.84
2	5.36	6.30	6.32	6.71	5.63

Note: The ARL_1 with the smallest values appear in bold.

Table 3: The examination of ARL_1 results for control charts depending on the Exponential distribution.

Shift	MA $B_1 = 3.338$	EWMA $B_2 = 3.78$	EEWMA $B_3 = 4.26$	MA-EEWMA $B_4 = 2.31$	MA-EEWMA Sign $B_5 = 19.24$
0	370.64	370.57	370.38	370.54	370.6
0.05	253.22	245.88	232.56	220.31	205.54
0.1	179.68	156.71	143.8	140.23	125.92
0.25	79.73	51.3	48.7	44.89	32.51
0.5	31.41	20.47	20.22	18.43	14.78
0.75	16.93	13.75	13.64	12.67	11.09
1	10.23	10.97	10.87	10.88	10.52
1.5	6.35	7.24	8.1	8.12	7.22
2	4.54	5.83	6.56	6.72	5.82

Note: The ARL_1 with the smallest values appear in bold.

Table 4: The examination of ARL_1 results for control charts depending on the Gamma distribution

Shift	MA $B_1 = 1.512$	EWMA $B_2 = 3.58$	EEWMA $B_3 = 4.162$	MA-EEWMA $B_4 = 2.42$	MA-EEWMA Sign $B_5 = 18.68$
0	370.35	370.72	370.63	370.59	370.61
0.05	344.03	327.93	325.44	315.55	308.04
0.1	320.39	316.93	275.95	259.67	228.67
0.25	239.48	226.45	222.96	203.42	198.51
0.5	141.71	124.97	100.98	90.62	83.34
0.75	89.61	70.98	54.49	50.55	44.48
1	40.75	30.99	18.5	18.85	18.76
1.5	4.52	2.41	2.11	4.77	4.79
2	2.52	1.52	1.5	2.61	2.60

Note: The ARL_1 with the smallest values appear in bold.

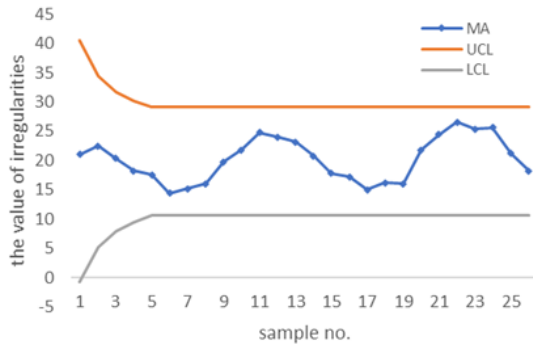


Figure 1: The potential of the MA chart to identify alterations related to nonconformities of printed circuit board data.

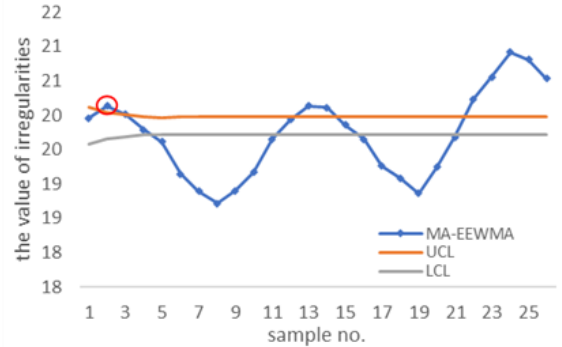


Figure 4: The potential of the MA-EEWMA chart to identify alterations related to nonconformities of printed circuit board data.

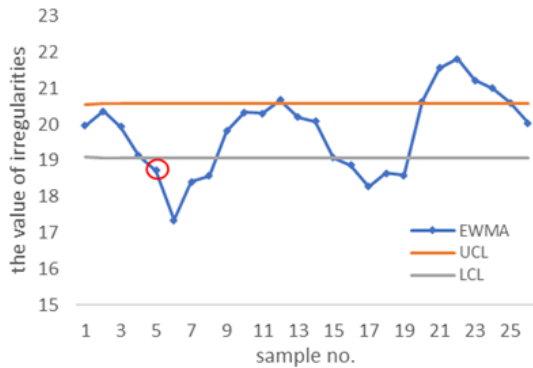


Figure 2: The potential of the EWMA chart to identify alterations related to nonconformities of printed circuit board data.

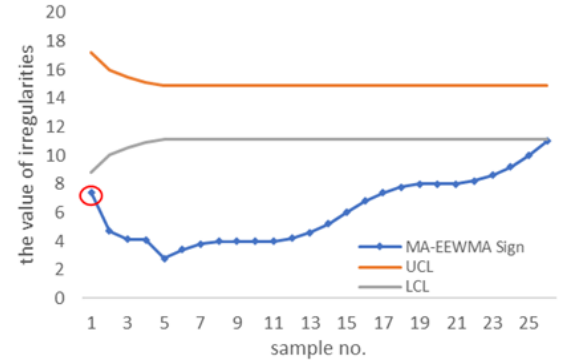


Figure 5: The potential of the MA-EEWMA Sign chart to identify alterations related to nonconformities of printed circuit board data.

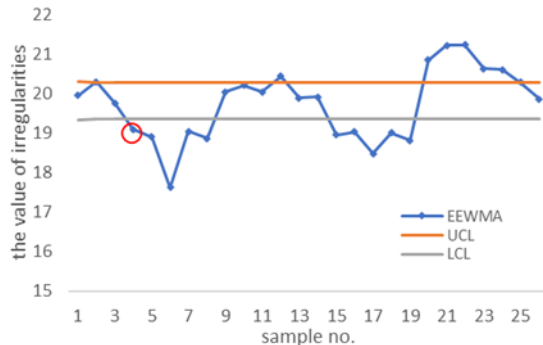


Figure 3: The potential of the EEWMA chart to identify alterations related to nonconformities of printed circuit board data.

Table 5: Inclusive efficacy of control charts.

Distribution	Charts	EQL	PCI
Normal	MA	7.20	1.27
	EWMA	6.68	1.18
	EEWMA	6.34	1.12
	MA-EEWMA	7.07	1.25
	MA-EEWMA Sign	5.66	1.00
Laplace	MA	21.02	2.35
	EWMA	12.62	1.41
	EEWMA	12.20	1.36
	MA-EEWMA	11.45	1.28
	MA-EEWMA Sign	8.96	1.00
Exponential	MA	7.50	1.01
	EWMA	7.65	1.03
	EEWMA	8.13	1.10
	MA-EEWMA	8.06	1.09
	MA-EEWMA Sign	7.41	1.00
Gamma	MA	18.43	1.64
	EWMA	14.64	1.30
	EEWMA	11.41	1.01
	MA-EEWMA	11.91	1.06
	MA-EEWMA Sign	11.26	1.00

Note: The smallest quantities are highlighted in bold.

This may be extremely helpful in formulating the out-of-control action plans offering have to include control charts. We plan to offer the proposed chart alongside the MA, EWMA, EEWMA, and MA-EEWMA charts, assuming a normal distribution with a mean of 19.846, a standard deviation of 7.164, with a significance level indicating normality with a p -value equal to 0.65.

Figures 1–5 display the results, which illustrate that the MA-EEWMA Sign control chart can identify the first sample rapidly. On the second, a change was discovered by the MA-EEWMA chart; on the fourth, by the EEWMA; and on the fifth, by the EWMA chart. Finally, the MA chart detected no change. The mentioned detections clearly indicate that the suggested design exhibits greater capability for specific reason detection. As a result, the recommended charts strategy may be effectively employed for tracking information within the workflow for more accurate alerting.

4 Conclusions

In order to improve ARL-based operating mean monitoring tools, this work introduces the MA-EEWMA Sign control chart, which is based on the nonparametric Sign. The study concludes that this suggested chart circumvents the problem that regular parametric control graphs present and finds application in cases when the observed distribution is either stubborn or unreliable. It serves various practical applications. With the lowest ARL_1 for minor to moderate shifts across all distributional settings, it turns out that the suggested chart is the best control chart after all.

Though EEWMA excels with the Gamma distribution, the MA chart outperforms the others when it comes to detecting big shifts. In addition, the suggested chart outperformed around the board throughout the distributions when measured using overall performance criteria based on EQL and PCL values. Parametric control charts aren't as effective as the suggested chart, though. Results from testing with actual data sets showed that the given chart worked well for finding changes quickly. A nonparametric MA-EEWMA sign chart serves as a powerful replacement while details about data and the process distribution are lacking. For added robustness, they may utilize the proposed nonparametric chart to boost their capacity to detect minor variations in the process.

However, the lengthy simulation duration is one of the study's weaknesses. Quality practitioners may decide that the suggested chart is the best way to control the abnormal process. Furthermore, while Sign statistics are the nonparametric statistic under scrutiny in this study, there are a number of nonparametric statistics which may enhance the effectiveness of a control chart, including Sign-Rank, which will be further examined in subsequent studies.

Appendix

This section estimates the mean and variance of the suggested MA-EEWMA sign statistics.

$$(MA-EE)_{S_k} = \begin{cases} \frac{S_k + S_{k-1} + S_{k-2} + \dots}{k}, & k < w \\ \frac{S_k + S_{k-1} + \dots + S_{k-w+1}}{w}, & k \geq w \end{cases}$$

where S_k follow the binomial distribution.

For $k < w$

$$E(MA-EE)_{S_k} = \frac{1}{k} \sum_{j=1}^k E(S_k) = \frac{1}{k} knp = np$$

$$V(MA-EE)_{S_k} = \frac{1}{k} \sum_{j=1}^k V(S_k) = \frac{1}{k^2} knpq = \frac{1}{k} npq V(EE_k)$$

For $k \geq w$

$$E(MA-EE)_{S_k} = \frac{1}{w} \sum_{j=k-w+1}^k E(S_k) = \frac{1}{w} wnp = np$$

$$V(MA-EE)_{S_k} = \frac{1}{w} \sum_{j=k-w+1}^k V(S_k) = \frac{1}{w^2} wnpq = \frac{1}{w} npq V(EE_k)$$

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Author Contributions

K.T.: conceptualization, methodology, data analysis, data curation, funding acquisition, project administration, writing an original draft; S.S.: prove and validate, investigation, reviewing and editing; All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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