Research Article

Design of Nonparametric Extended Exponentially Weighted Moving Average – Sign Control Chart

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Received: 3 August 2023; Revised: 19 September 2023; Accepted: 9 October 2023; Published online: 4 December 2023

Abstract
The present research introduces the EEWMA-Sign chart, which incorporates the extended exponentially weighted moving average control chart with the sign control charts to detect small changes in procedures. This is a nonparametric control chart that can overcome the constraints imposed by normal assumptions. The average run lengths serve as supporting examinations for comparing the effectiveness of a monitoring scheme to the EEWMA and EWMA control charts via Monte Carlo Simulation. Besides a specific range of shift sizes, the expected ARL (EARL) remains an instrument to assess the efficiency of control charts. The overall result demonstrates that the proposed chart is the most suitable control chart for detecting small shifts between Normal, Lognormal, and Laplace distributional scenarios. Nonetheless, the EWMA chart recognizes large shifts more efficiently than others. Adapting the proposed control chart to the flow width dataset produced results consistent with the research findings.

Keywords: Average Run Length, Detection, Extended Exponentially Weighted Moving Average control chart, Mean process, Sign test

1 Introduction

Statistical process control (SPC) utilizes technology to observe and regulate manufacturing processes. Through product examinations and computational tests, SPC leads to various machines and gadgets that provide quality data. To control the process, data is collected, analyzed, and monitored. Statistical process control is an easy way to promote continual enhancement. Directors may guarantee that a procedure achieves its full potential by continually tracking and overseeing it, leading to regular, excellent manufacturing. Control charts, as well as a focus on continuous improvement, constitute crucial instruments in SPC. A control chart is a graphic depiction of the upper control limit (UCL), the center line (CL), and the lower control limit (LCL). Suppose the displayed statistic collapses within the control limits. In that case, the process is steady and under control, whereas any point exterior the control limits suggests that the process is out of control. Traditional control charts were developed in earlier times and were effective at detecting both small and large shifts in a procedure attribute. Based on current data, Shewhart [1] became the initial researcher to use the control charting approach, therefore useless in detecting small to moderate shifts. The exponentially weighted moving average (EWMA) [2], cumulative sum (CUSUM) [3], and modified exponentially weighted moving average (MEWMA) [4] charts employed both previous and present data, making them more sensitive to small and moderate changes. Naveed *et al.*, [5] establish an extended exponentially weighted moving average control chart (EEWMA) with unbiased and low variance. Regarding detecting shifts quickly, the EEWMA control chart outperforms the EWMA and Shewhart control charts.

On the other hand, successful monitoring is a crucial part of the upgrade procedure. The average run length (ARL) is an extensive metric for determining the efficacy of the proposed control chart. Aside from a particular range of shift sizes, the expected ARL (EARL) is still used to determine the effectiveness of control charts.

In general, traditional control charts (parametric control charts) are generated using a particular distributional assumption, most commonly the normal distribution. In contrast, many actual-life data embraces are non-normal or lack knowledge of the distribution. For resolving these constraints, nonparametric control charts are a viable and powerful solution with the following advantages [6]: convenience, no requirement to assume a specific parametric distribution for the underlying procedure, greater robustness and outlier resistance, and no necessity for estimating the variance for setting up charts for the location parameter. Many researchers use the Sign statistic with other control charts, such as an EWMA-Sign control chart designed by Yang *et al.*, [7]. The proposed chart's ARL performs admirably in monitoring the deviation from the procedure's goal. Yang and Cheng [8] developed a new nonparametric CUSUM mean chart using the sign statistic which showed better detection ability in small shifts. Lu [9] developed the generally weighted moving average (GWMA) Sign control chart to enhance the ability to detect small process shifts. These findings display that the nonparametric GWMA sign chart proves more efficient than the parametric GWMA chart based on the smaller out-of-control ARL, and it should be used instead when process information is lacking. In 2020, Aslam *et al.*, [10] presented a modified exponentially weighted moving average control chart combined with the sign statistic (MEWMA-Sign) via the average run length serving as efficiency evaluations. The results revealed that the MEWMA-Sign chart proved better than the EWMA-Sign chart at recognizing changes. Taboran and Sukparungsee [11] investigate the MEWMA-Sign control chart for monitoring procedure mean under non-normal distributions, and the findings show that the chart exceeds EWMA-Sign in detecting a tiny shift determined by the least out-of-control ARL. Petcharat and Sukparungsee [12] proposed the MEWMA-Sign Rank control chart which extended work from [11] and found that the MEWMA-Sign Rank chart outperformed for moderate changes.

In this study, the EEWMA chart based on the Sign statistics is being used to create a new control chart to evaluate the process mean effectively. This provides the nonparametric control chart, which is utilized for processes where the observations' distribution is unknown or when the parameter cannot be estimated. The average run length (ARL) and the expected ARL (EARL) [13] of the proposed (EEWMA-Sign) chart have been computed and compared with existing charts when the process follows symmetric distributions, which the mean and the median are the same and any of these can serve as a measure of the typical, Normal(0,1) and Lognormal(0,1) whereas skew distribution is Laplace(1,1) [14]. Furthermore, using an actual data set of flow width measurements in the hard-bake process was feasible.

2 Design of Proposed Control Chart

In the following section, we will construct an extensive control chart for recognizing demand shifts. EWMA and EEWMA are parametric control charts that are based on the concepts of normality, dependence, and variance homogeneity. On the other hand, when the distribution of a process is not assumed to be normal, a nonparametric control chart, including Sign and the proposed EEWMA-Sign control charts, is recommended as an alternative to a regular control chart.

Assume $X_1, X_2, \ldots, X_t, \ldots$ are independent and identically distributed random variables drawn from a normal sample with mean μ and variance σ_2. The control chart layout established essentially follows.

2.1 *Exponentially Weighted Moving Average (EWMA) control chart*

Roberts [2] introduced the EWMA statistic with a smoothing parameter α; $(0 < \alpha \leq 1)$, as shown in Equation (1):

$$EWMA_t = \alpha X_t + (1-\alpha)EWMA_{t-1}. \quad (1)$$

The initial value of $EWMA_0$ is usually given to equal μ. The mean and asymptotic variance, when $t \rightarrow \infty$, are shown in Equations (2) and (3) as follows:

$$E(EWMA_t) = \mu \quad (2)$$

$$V(EWMA_t) = \sigma^2 \left[\frac{\alpha}{2-\alpha} \right]. \quad (3)$$

The EWMA chart's control limits are shown in Equation (4):

$$UCL/LCL = \mu \pm C_1 \sigma \sqrt{\frac{\alpha}{2-\alpha}} \quad (4)$$

where μ and σ represents the average and standard deviation of the processes under consideration, C_1 is the initial steady of the EWMA control chart's suitable limit.

2.2 Extended Exponentially Weighted Moving Average (EEWMA) control chart

Naveed *et al.*, [5] designed the EEWMA control chart to be extremely useful in detecting a rapid shift in the mean. The EEWMA statistic is defined in Equation (5) as follows:

$$EEWMA_t = \alpha_1 X_t - \alpha_2 X_{t-1} + (1-\alpha_1+\alpha_2) EEWMA_{t-1} \quad (5)$$

where α_1 and α_2 are smoothing parameters ranging from 0 to 1, which $0 < \alpha_1 \le 1$ and $0 \le \alpha_2 < \alpha_1$. The values of $EEWMA_0$ and X_0 are taken as the target mean. The average and the asymptotic variance of $EEWMA_t$ when $t \rightarrow \infty$ are defined in Equations (6) and (7):

$$E(EEWMA_t) = \mu \quad (6)$$

and

$$V(EEWMA_t) = \sigma^2 \left[\frac{\alpha_1^2 + \alpha_2^2 - 2\alpha_1\alpha_2(1-\alpha_1+\alpha_2)}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2} \right]. \quad (7)$$

The control limits of the EEWMA chart are as follows in Equation (8) below,

$$UCL/LCL = \mu \pm C_2 \sigma \sqrt{\frac{\alpha_1^2 + \alpha_2^2 - 2\alpha_1\alpha_2(1-\alpha_1+\alpha_2)}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2}} \quad (8)$$

where μ and σ represents the average and standard deviation of the processes under consideration, C_2 is the initial steady of the EEWMA control chart's suitable limit.

2.3 *Sign statistics*

Suppose X_{jt}, $j = 1,2,...,n$ and $t = 1,2,3,...$, denote the t^{th} observation in the j^{th} logical subgroup of size n.

Let the known target value be monitored is T, then the difference between the observations and the target value, i.e. $X_{jt} - T$ within groups, can be denoted by Equation (9) as follows:

$$Y_{jt} = X_{jt} - T, \; t = 1,2,3,..., \; j = 1,2,...,n. \quad (9)$$

The Sign statistic S_t can be defined as Equation (10):

$$S_t = \sum_{j=1}^{n} I_{jt}. \quad (10)$$

Equation (10) I_{jt} can be elaborated as Equation (11):

$$I_{jt} = \begin{cases} 1, & Y_{jt} > 0 \\ 0, & \text{otherwise} \end{cases}. \quad (11)$$

Then, the Sign statistic is the total number of observations following the binomial distribution with a parameter (n, $p_0 = 0.5$)for the control case. The value of $p = P(Y > 0)$ is the process proportion which $p = p_0 = P(Y \le T) = P(Y > T) = 0.5$ is in the control process. On the other hand, the process is out of control when $q_0 \neq 0.5$.

2.4 *Extended Exponentially Weighted Moving Average – Sign (EEWMA-Sign) control chart*

The proposed EEWMA-Sign control chart was created by combining the EEWMA control chart and the Sign test. The EEWMA-Sign control design statistic is described this way in Equation (12):

$$EEWMA_{St} = \alpha_1 S_t - \alpha_2 S_{t-1} + (1-\alpha_1+\alpha_2) EEWMA_{St-1} \quad (12)$$

where α_1 and α_2 are smoothing parameters ranging from 0 to 1, which $0 < \alpha_1 \le 1$ and $0 \le \alpha_2 < \alpha_1$. Adopting the starting value, $EEWMAS_0 = np_0$. The average and variance of EEWMA-Sign are given as

Equations (13) and (14). The derivation of mean and variance are shown in the Appendix.

$$E(EEWMA_{St}) = np_0 \quad (13)$$

and

$$V(EEWMA_{St}) = np_0 q_0 \left[\frac{\alpha_1^2 + \alpha_2^2 - 2\alpha_1\alpha_2(1-\alpha_1+\alpha_2)}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2} \right]. \quad (14)$$

The control limits of the EEWMA-Sign chart are as follows in Equation (15) below,

$$UCL/LCL = np_0 \pm C_3\sqrt{Var(EEWMA_{St})} \quad (15)$$

where C_3 is the initial steady of the EEWMA-Sign control chart's suitable limit.

3 Performance Measurement

In the manufacturing industry, change detection tests are frequently used. The average run length (ARL) serves as one of the assessment techniques for identifying process changes. It displays the average data points until the signal goes out. The average number of observations (or tracking points) before a signal point declines outside the control limits, whereas the process is in control and referred to as the in-control ARL (ARL_0). On the other hand, the average number of findings needed before detecting a shift in the mean once the process is out of control is referred to as the out-of-control ARL (ARL_1). It is crucial to detect a change in the process as quickly as feasible, which implies that ARL_1 there should be more to guarantee that the control chart is efficient.

In the following step, the ARL attributes of all control charts were evaluated using Monte Carlo simulation: To begin, select a random sample from any given distribution. After that, compute the proposed charting statistic and consider "*C*" at a value of ARL_0 = 370. The control limit is then calculated, and the values statistic is run. Finally, iterate 100,000 times (N) to compute the ARL.

The ARL is described in Equation (16) below:

$$ARL = \frac{\sum_{t=1}^{N} RL_t}{N} \quad (16)$$

where RL_t refers to the number of samples required before the method becomes unmanageable for the initial time, N is the number of repeated t trials and the amount of data simulations.

Furthermore, a method for evaluating the performance of control charts over a specific range of shift sizes is the expected ARL (EARL) [12], [13], which takes the EARL into account as deciding the overall range of shifts (ψ_1, ψ_2), regarding the chart with the lowest EARL value being the most efficient. The EARL solution is identified as follows in Equation (17):

$$EARL = \frac{1}{\psi_2 - \psi_1}\int_{\psi_1}^{\psi_2} ARL(\psi)d\psi \quad (17)$$

where ψ_1 and ψ_2 represent the lower and upper bounds of the shift, respectively. $ARL(\psi)$ indicates the ARL value of a chart for the specified shift.

4 Numerical Results

The following section offers the proposed chart assessment founded on the previously explained performance indicators from 100,000 repetitions with 5 and 10 subgroups in Monte Carlo simulations under ARL0 = 370. Small subgroups were used because they are frequently used in process practice and take less time and money. The run length characteristics for all charts were obtained via simulations with α = 0.10, 0.25 in the EWMA chart while used α_1 = 0.10, α_2 = 0.03 and α_1 = 0.25, α_2 = 0.10 in the EEWMA and proposed charts for Normal(0,1), Lognormal(0,1), and Laplace(1,1) distributions with specific shifts of 0, 0.02, 0.05, 0.1, 0.15, 0.2, 0.3. 0.5, 1.0, and 1.5. Furthermore, we evaluate the performance of the proposed chart (EEWMA-Sign) to that of existing charts, regarding the chart with the smallest ARL_1 announced to be the most efficient. The bold values indicate that the chart executed better in terms of minimizing ARL_1.

The numerical findings of the proposed control chart when the smoothing parameter 0.1, 0.25 is varied for the Normal(0,1), Lognormal(0,1), and Laplace(1,1) distributions. We discovered that as the value of the smoothing parameter increased, the coefficient control limit of EWMA and EEWMA was also raised, except for EEWMA-Sign.

In addition, the results obtained under Normal(0,1) distribution displayed in Table 1 by subgroup n = 5

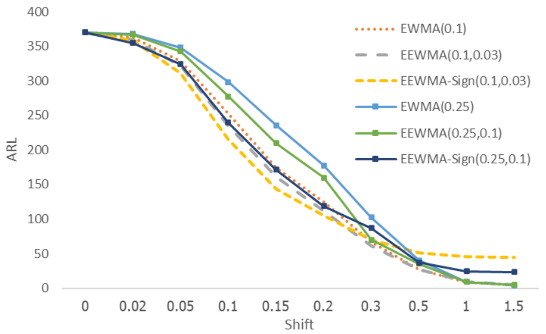


Figure 1: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Normal distribution with n = 5.

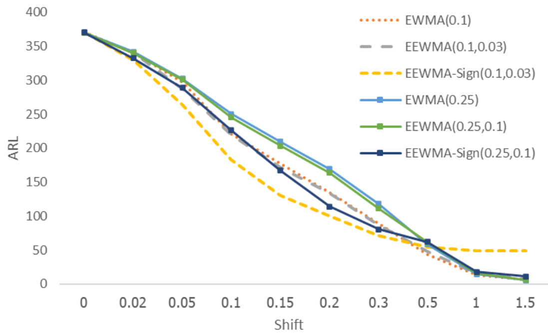


Figure 3: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Lognormal distribution with n = 5.

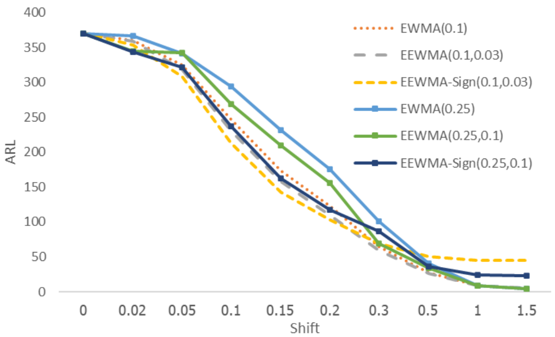


Figure 2: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Normal distribution with n = 10.

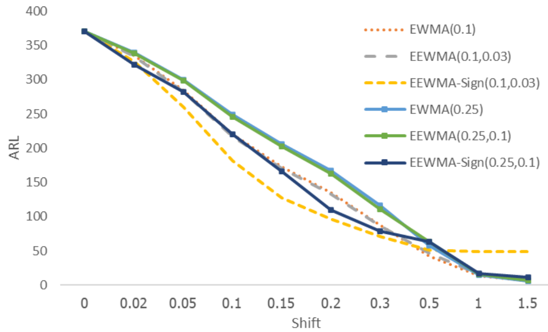


Figure 4: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Lognormal distribution with n = 10.

demonstrates that the proposed chart (EEWMA-Sign) $\alpha_1 = 0.10$, $\alpha_2 = 0.03$ has slightly more excellent detectability in shifts 0.02–0.2. In contrast, the EEWMA control chart outperforms in shifts 0.3–0.5, and the EWMA control chart $\boldsymbol{\alpha} = 0.10$ performs slightly better than the other charts in shifts 1–1.5. However, a similar result is obtained when $\alpha_1 = 0.25$, $\alpha_2 = 0.10$ in the proposed EEWMA-Sign, the EEWMA charts, and the EWMA with $\alpha = 0.25$. Table 2 reveals the results from the subgroup $n = 10$, and the proposed chart depicts a similar incident with $n = 5$. The EEWMA sign is superb for detecting shifts 0.02–0.2; alternatively, the EEWMA and EWMA will recognize shifts 0.3–0.5 and 1–1.5, respectively, in all smoothing parameters. The performance can be seen graphically in Figures 1 and 4.

Tables 3 and 4 display a discovery from the Lognormal(0,1) distribution when subgroups are 5 and 10, respectively. When $\alpha_1 = 0.10$, $\alpha_2 = 0.03$ and coefficient of control limit 7.914 were used in subgroup $n = 5$, the ARL_1 of the EEWMA-Sign chart was lower than the other control charts with change levels of 0.02, 0.05, 0.1, 0.15, 0.2, and 0.3. Although the change parameter level was set to 0.5, 1.0, and 1.5, the EWMA control chart with $\alpha = 0.10$ proved the most effective at detecting changes. Nevertheless, the same result happens when $\alpha_1 = 0.25$, $\alpha_2 = 0.10$ in the proposed EEWMA-Sign, the EEWMA charts, as well as the EWMA with $\alpha = 0.25$. Furthermore, the numerical findings via subgroups $n = 10$ generate the same outcome as subgroups $n = 5$. Figures 2 and 5 indicate the outcome graphically.

The results of a Laplace(1,1) distribution in Table 5 with subgroups n = 5 show that the proposed EEWMA-Sign control chart with a coefficient value of 7.838 at $\alpha_1 = 0.10$, $\alpha_2 = 0.03$ exhibited a ridiculously

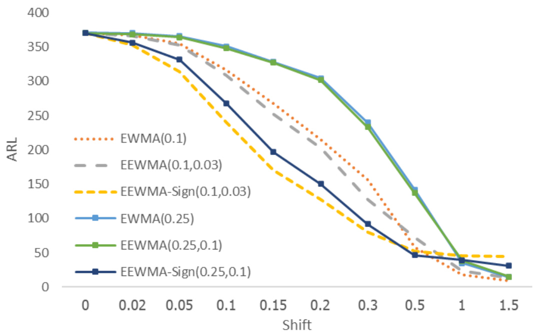


Figure 5: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Laplace distribution with n = 5.

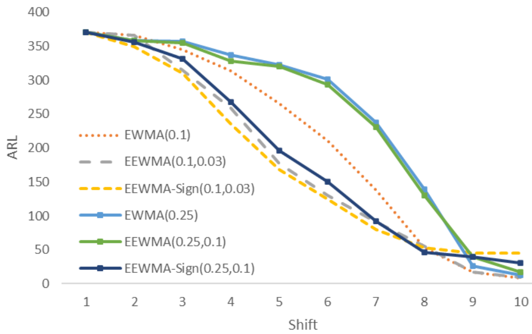


Figure 6: ARL curves of EWMA, EEWMA, and EEWMA-Sign depend on the smoothing parameter for the Laplace distribution with n = 10.

low ARL_1 when the detecting shift parameter was set at shifts ranging from 0.02–0.5, excluding 1.0 and 1.5, where the EWMA control chart exceeded. However, the same result is obtained when α = 0.25. Moreover, in all smoothing parameters, the computational results obtained through subgroups n = 10 produce similar results as subgroups n = 5 as shown in Table 6. Figures 3 and 6 clearly show the performance.

Besides that, we presented the findings of an evaluation of EARL values for period shifts. Table 7 depicts that for small shift sizes, the EARL values of the proposed chart are always less than the other charts for all distributions (Normal distribution = 2.83, Lognormal distribution = 2.67, and Laplace distribution = 3.19). However, if we examine a moderate to large shift in the approach from Table 8, we observe that the EARL of the EWMA chart are slightly fewer than the other charts in Lognormal (EARL = 13.96) and Laplace distributions (EARL = 34.15), with the exception of the Normal distribution (EARL = 9.40), where the EEWMA chart outperforms the others.

5 Demonstrative Case

We will demonstrate the proposed chart, as well as the EWMA, EEWMA and EEWMA-Sign control charts with flow width measurements (microns) for the Hard-bake process as n = 5, m = 45 by Montgomery [15] under normal distribution with mean 1.5318, standard deviation = 0.1435, and the significance to be a normal distribution with p-value = 0.82. The results demonstrated in Figure 7 and show that the EEWMA-Sign control chart can detect quickly in the first sample. The EEWMA chart then identified a change in the sixth sample while the EWMA chart detected a change in the fifthteen sample.

Table 1: ARL1 values for the Normal(0,1) distribution of EWMA, EEWMA, and EEWMA-Sign with n = 5

Shift	α = 0.10	α_1 = 0.10, α_2 = 0.03		α = 0.25	α_1 = 0.25, α_2 = 0.10	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	C_1 = 2.702	C_2 = 1.940	C_3 = 7.783	C_1 = 2.899	C_2 = 2.147	C_3 = 4.882
0	370.01	370.13	370.19	370.45	370.34	370.23
0.02	363.28	359.36	**358.40**	368.00	366.91	**355.33**
0.05	328.94	322.04	**311.82**	348.39	342.85	**324.12**
0.1	252.96	235.56	**215.72**	298.49	277.38	**239.93**
0.15	174.65	160.98	**143.61**	235.74	210.21	**171.39**
0.2	124.87	111.64	**104.65**	177.78	160.08	**118.93**
0.3	65.94	**60.97**	70.07	102.51	**69.85**	87.12
0.5	27.37	**26.76**	50.63	40.49	**35.26**	37.28
1	**8.75**	9.18	45.12	**9.24**	9.25	24.52
1.5	**4.85**	5.16	44.45	**4.17**	4.51	23.45

Note: The bold is minimal of ARL_1 of the control chart.

Table 2: ARL1 values for the Normal(0,1) distribution of EWMA, EEWMA, and EEWMA-Sign with $n = 10$

Shift	$\alpha = 0.10$	$\alpha_1 = 0.10, \alpha_2 = 0.03$		$\alpha = 0.25$	$\alpha_1 = 0.25, \alpha_2 = 0.10$	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	$C_1 = 2.703$	$C_2 = 2.706$	$C_3 = 11.017$	$C_1 = 2.898$	$C_2 = 2.912$	$C_3 = 9.378$
0	370.14	370.54	370.69	370.34	370.21	370.31
0.02	360.16	359.13	**353.21**	367.21	344.77	**343.38**
0.05	324.87	318.99	**308.29**	341.43	342.34	**322.35**
0.1	247.24	230.74	**213.18**	293.82	269.37	**237.34**
0.15	174.03	158.17	**142.84**	232.24	210.11	**163.06**
0.2	122.65	109.63	**103.66**	176.03	155.76	**117.17**
0.3	64.75	**59.61**	69.44	101.42	**69.16**	87.09
0.5	27.21	**26.23**	50.19	40.24	**34.39**	35.88
1	**8.73**	9.14	45.17	**9.04**	9.15	23.70
1.5	**4.79**	5.22	44.47	**4.14**	4.40	22.62

Note: The bold is minimal of ARL_1 of the control chart.

Table 3: ARL1 values for the Lognormal(0,1) distribution of EWMA, EEWMA, and EEWMA-Sign with $n = 5$

Shift	$\alpha = 0.10$	$\alpha_1 = 0.10, \alpha_2 = 0.03$		$\alpha = 0.25$	$\alpha_1 = 0.25, \alpha_2 = 0.10$	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	$C_1 = 14.181$	$C_2 = 11.410$	$C_3 = 7.914$	$C_1 = 14.300$	$C_2 = 11.860$	$C_3 = 4.990$
0	370.83	370.13	370.35	370.86	370.26	370.38
0.02	340.78	340.21	**329.21**	341.83	340.53	**332.56**
0.05	296.56	287.07	**263.58**	302.37	301.26	**289.25**
0.1	220.96	219.84	**183.50**	250.19	245.23	**226.65**
0.15	177.34	171.83	**130.76**	209.52	203.44	**167.11**
0.2	134.57	134.00	**100.97**	169.58	163.59	**113.96**
0.3	89.02	86.67	**71.89**	117.79	111.09	**80.72**
0.5	**42.88**	47.42	54.25	**58.83**	61.79	62.33
1	**13.38**	15.78	49.21	**14.67**	16.35	17.85
1.5	**6.32**	7.54	49.02	**5.55**	6.53	11.42

Note: The bold is minimal of ARL_1 of the control chart.

Table 4: ARL1 values for the Lognormal(0,1) distribution of EWMA, EEWMA, and EEWMA-Sign with $n = 10$

Shift	$\alpha = 0.10$	$\alpha_1 = 0.10, \alpha_2 = 0.03$		$\alpha = 0.25$	$\alpha_1 = 0.25, \alpha_2 = 0.10$	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	$C_1 = 14.230$	$C_2 = 15.910$	$C_3 = 11.470$	$C_1 = 14.304$	$C_2 = 16.206$	$C_3 = 5.937$
0	370.44	370.04	370.24	370.61	370.35	370.44
0.02	334.74	334.47	**326.52**	339.71	338.06	**322.20**
0.05	284.19	282.18	**260.16**	299.55	298.78	**281.66**
0.1	220.19	217.02	**181.98**	248.61	245.13	**220.73**
0.15	172.38	170.73	**127.23**	205.89	202.48	**166.59**
0.2	135.31	132.52	**96.30**	167.10	163.28	**110.12**
0.3	88.19	86.32	**70.85**	116.40	110.40	**78.34**
0.5	42.75	46.86	51.44	**58.02**	63.25	63.09
1	**13.37**	15.67	49.07	**14.55**	15.88	17.41
1.5	**6.36**	7.59	48.81	**5.56**	6.76	11.37

Note: The bold is minimal of ARL_1 of the control chart.

Table 5: ARL1 values for the Laplace(1,1) distribution of EWMA, EEWMA, and EEWMA-Sign with n = 5

Shift	$\alpha = 0.10$	$\alpha_1 = 0.10, \alpha_2 = 0.03$		$\alpha = 0.25$	$\alpha_1 = 0.25, \alpha_2 = 0.10$	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	$C_1 = 4.010$	$C_2 = 2.960$	$C_3 = 7.838$	$C_1 = 4.715$	$C_2 = 3.674$	$C_3 = 4.911$
0	370.84	370.08	370.53	370.92	370.35	370.32
0.02	366.98	366.11	**352.29**	370.21	368.59	**356.12**
0.05	355.17	352.87	**313.99**	365.65	364.25	**331.43**
0.1	315.77	308.61	**239.78**	350.87	347.98	**267.65**
0.15	267.38	252.44	**170.96**	328.27	326.91	**196.59**
0.2	214.72	203.02	**127.89**	304.36	301.52	**150.06**
0.3	155.85	127.58	**80.38**	239.78	233.40	**91.81**
0.5	57.75	71.67	**52.64**	142.23	136.80	**46.37**
1	**18.07**	23.79	45.16	**34.96**	38.10	39.38
1.5	**9.62**	13.66	45.00	**14.19**	14.84	31.24

Note: The bold is minimal of ARL_1 of the control chart.

Table 6: ARL1 values for the Laplace(1,1) distribution of EWMA, EEWMA, and EEWMA-Sign with n = 10

Shift	$\alpha = 0.10$	$\alpha_1 = 0.10, \alpha_2 = 0.03$		$\alpha = 0.25$	$\alpha_1 = 0.25, \alpha_2 = 0.10$	
	EWMA	**EEWMA**	**EEWMA-Sign**	**EWMA**	**EEWMA**	**EEWMA-Sign**
	$C_1 = 4.291$	$C_2 = 4.691$	$C_3 = 11.086$	$C_1 = 4.720$	$C_2 = 5.025$	$C_3 = 5.035$
0	370.58	370.23	370.83	370.13	370.68	37032
0.02	366.45	366.05	**349.10**	358.59	358.06	**356.12**
0.05	344.40	314.10	**310.41**	357.26	354.89	**331.43**
0.1	313.62	259.18	**234.88**	336.54	327.96	**267.65**
0.15	265.87	176.71	**168.10**	321.88	320.62	**196.59**
0.2	210.24	129.79	**124.15**	301.13	293.17	**150.06**
0.3	138.29	91.32	**79.82**	237.38	230.41	**91.81**
0.5	52.80	55.55	**52.64**	139.29	130.14	**46.37**
1	**17.29**	17.47	45.08	**26.55**	39.35	39.38
1.5	**8.70**	9.36	45.00	**13.29**	17.02	31.24

Note: The bold is minimal of ARL_1 of the control chart.

6 Discussion and Conclusions

This paper aims to present an EEWMA control chart based on the nonparametric Sign to develop better process mean monitoring strategies using the ARL, known as the EEWMA-Sign control chart. The following proposed chart may be utilized when the observed distribution is inflexible or unknown, overcoming the limitation of traditional parametric control charts and being extremely helpful in numerous real-world scenarios. The outcomes exhibit that the proposed chart remains the most suitable control chart across all distributional locations via the smallest ARL_1 for small shifts. Despite this, the EWMA chart recognizes large shifts stronger than other charts. However, the performance of the proposed chart is better than parametric control charts. An examination of performance in real-world data applications demonstrated that the provided chart was suited to discovering shifts rapidly. In addition, because the nonparametric statistic investigated in this study is Sign statistics, presently are several nonparametric statistics that can improve the efficiency of a control chart, such as Sign-Rank and Arcsine, which the researchers will explore in more detail in future research.

7 Appendix

In this part, we estimate the mean and variance of the proposed EEWMA-sign statistics.

$$EEWMA_{S_t} = \alpha_1 S_t - \alpha_2 S_{t-1} + (1-\alpha_1+\alpha_2)EEWMA_{S_{t-1}}$$

$$EEWMA_{S_1} = \alpha_1 S_1 - \alpha_2 S_0 + (1-\alpha_1+\alpha_2)EEWMA_{S_0}$$

$$EEWMA_{S_2} = \alpha_1 S_2 - \alpha_2 S_1 + (1-\alpha_1+\alpha_2) EEWMA_{S_1}$$

Let $a = (1-\alpha_1+\alpha_2)$, $EEWMA_{S_2} = \alpha_1 S_2 - \alpha_2 S_1 + a\alpha_1 S_1 - a\alpha_2 S_0 + a^2 EEWMA_{S_0}$

Let $b = (a\alpha_1 - \alpha_2)$, $EEWMA_{S_2} = \alpha_1 S_2 + bS_1 - a\alpha_2 S_0 + a^2 EEWMA_{S_0}$

Then, $EEWMA_{S_t} = \alpha_1 S_t + bS_{t-1} + abS_{t-2} + a^2 bS_{t-3} + ... + a^{t-2} bS_1 - a^{t-1}\alpha_2 S_0 + a^t EEWMA_{S_0}$. (A)

Taking expectation of Equation (A):

$$E(EEWMA_{S_t}) = \alpha_1 E(S_t) + bE(S_{t-1}) + abE(S_{t-2}) + a^2 bE(S_{t-3}) + ... + a^{t-2} bE(S_1) - a^{t-1}\alpha_2 E(S_0) + a^t E(EEWMA_{S_0})$$

$$E(EEWMA_{S_t}) = np_0[(\alpha_1 - \alpha_2)\{1 + a + a^2 + ... + a^{t-2} + a^{t-1}\} + a^t].$$

We obtain: $E(EEWMA_{S_t}) = np_0\left[(\alpha_1-\alpha_2)\left\{\frac{1-a^t}{1-a}\right\} + a^t\right]$. Therefore, $E(EEWMA_{S_t}) = np_0$.

Taking variance of Equation (A):

$$\begin{aligned} V(EEWMA_{S_t}) &= \alpha_1 V(S_t) + bV(S_{t-1}) + abV(S_{t-2}) + a^2 bV(S_{t-3}) + ... + a^{t-2} bV(S_1) - a^{t-1}\alpha_2 V(S_0) + a^t V(EEWMA_{S_0}) \\ &= np_0 q_0\left[(\alpha_1^2+\alpha_2^2)\{1 + a^2 + a^4 + ... + a^{2(t-1)}\} - 2a\alpha_1\alpha_2\{1 + a^2 + a^4 + ... + a^{2(t-2)}\}\right] \\ &= np_0 q_0\left[(\alpha_1^2+\alpha_2^2)\left\{\frac{1-a^{2t}}{1-a^2}\right\} - 2a\alpha_1\alpha_2\left\{\frac{1-(a^2)^{t-1}}{1-a^2}\right\}\right]. \end{aligned}$$

Thus, $V(EEWMA_{St}) = np_0 q_0\left[\frac{\alpha_1^2+\alpha_2^2-2\alpha_1\alpha_2(1-\alpha_1+\alpha_2)}{2(\alpha_1-\alpha_2)-(\alpha_1-\alpha_2)^2}\right]$.

Table 7: Evaluation of chart performance based on EARL values for small shift sizes

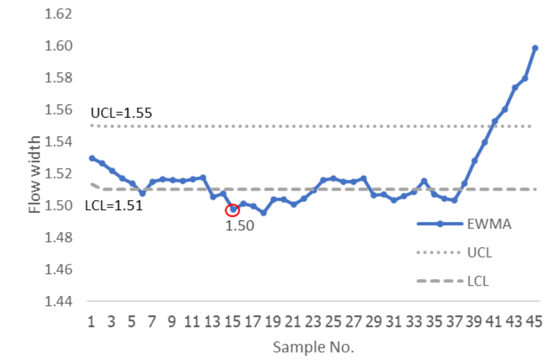
Distribution	Shift Sizes [0, 0.3]		
	EWMA	**EEWMA**	**EEWMA-Sign**
Normal	3.66	3.03	**2.83**
Lognormal	3.64	3.49	**2.67**
Laplace	6.53	6.42	**3.19**

Note: The bold is minimal of EARL of the control chart.

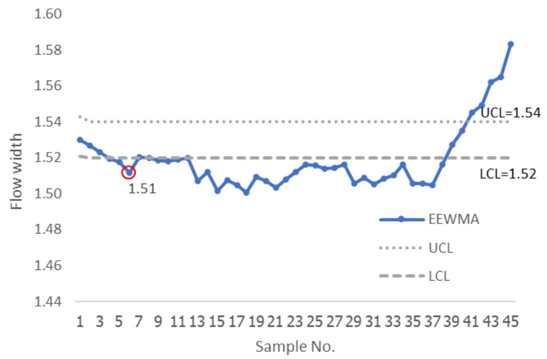
Table 8: Evaluation of chart performance based on EARL values for moderate to large shift sizes

Distribution	Shift Sizes [0.5, 1.5]		
	EWMA	**EEWMA**	**EEWMA-Sign**
Normal	9.58	9.40	28.87
Lognormal	13.96	15.50	19.71
Laplace	34.15	35.23	40.42

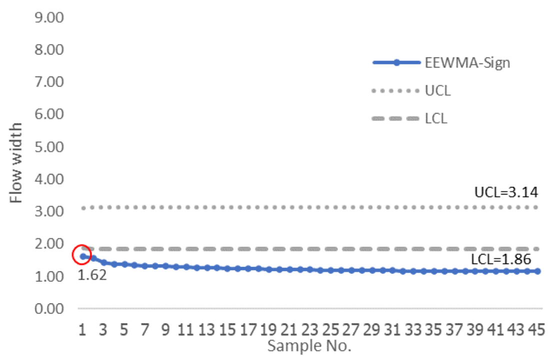
Note: The bold is minimal of EARL of the control chart.



(a)



(b)



(c)

Figure 7: The ability of ARL_1 of the EWMA chart (a), EEWMA chart (b), and EEWMA-Sign chart (c) to detect a change in flow width data.

Acknowledgments

We would like to express our gratitude to Thailand Science Research and Innovation, Ministry of Higher Education, Science, Research for supporting the research fund with Contract No. KMUTNB-FF-66-06.

Author Contributions

K.T.: investigation, methodology, data analysis, data curation, writing an original draft; Y.A. proves and validates; S.S.: conceptualization, investigation, funding acquisition, project administration, reviewing and editing; All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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