Research Article

Development of a New MEWMA – Wilcoxon Sign Rank Chart for Detection of Change in Mean Parameter

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Abstract
A nonparametric chart has been accepted and implemented for real world problems, especially, when the distribution of the population is unknown and the parameter could not be estimated. The nonparametric chart can overcome those limitations and it is user-friendly. Consequently, the objective of this research is to develop a new modified exponentially weighted moving average (MEWMA) chart based on Sign Rank statistics, namely MEWMA-SR, and to compare the performance of change detection with the EWMA and MEWMA charts and nonparametric EWMA based on the Sign (EWMA-Sign) and the Sign Rank (EWMA-SR), and MEWMA-Sign charts. The efficiency measurement of charts is commonly performed by average run length (ARL) divided into two states; in control ARL (ARL_0) and out of control ARL (ARL_1). The numerical results were carried out by Monte Carlo simulation with 10^5 replication and the best performance of the chart is considered by the minimum value of ARL_1. The proposed chart outperforms when a subgroup is small and a magnitude of changes is moderate for Laplace, otherwise the EWMA-SR is superior to small changes. When the observations are from lognormal the MEWMA-SR performs better than EMWA-SR and other charts for moderate to large changes for all sizes of subgroups. Furthermore, the proposed chart is applied to real data set as the S&P 500 index and shows the best performance in detecting a change.

Keywords: Nonparametric control chart, Wilcoxon sign rank, Modified exponentially weighted moving average, Average run length

1 Introduction

Statistical quality control (SQC) is used to control, monitor, detect, and improve the quality of the process. There are 7 types of quality control tools by statistical quality control, including check sheets, histogram, Pareto diagram, cause and effect diagram or fish-bone diagram, scatter diagram, and control chart, which are powerful statistical tools to detect changes in the mean process. The most process is the control chart because it can be clearly displayed in a graph and is now widely used. The main objective of using a control chart is the speed at which it can quickly detect changes in the mean or other process parameters. The lowest false alarm rate should be given when the process is in control and the highest true alarm rate should be given when the process is out of control. Several applications are using statistical process control charts; i.e., computer intrusion [1], industrial manufacturing [2], health science statistics [3], [4], financial business [5], environmental statistics [6], petroleum and meteorology [7].

The control charts can be divided into two types: variables control chart, a chart that is used to control the production process when quality characteristics can be measured, such as the diameter of specimens, average length of the product and average lifetime of the battery, etc. These quality characteristics are quantitative as shown in numeric. The popular control charts for the first type are $\overline{X}$ chart, range chart (R chart), and standard deviation chart (S chart), which were first proposed by Shewhart [8] for processes with normal distribution and good efficacy in detecting large-scale changes [9]. The second type is the attributes control chart, which is used for the detection of the number of nonconformities products, number of defects, or proportion of defects. Examples of this type of control chart are the proportion of defects control chart (p chart), the number of the defective product (np chart), the number of conforming products (c chart), and the number of conforming products per unit chart (u chart).

However, the flaw of the standard control chart is not focused on previous data. Then, it is unable to remember the small past changes that collected over time, which this characteristic is called "memoryless". In the past few decades, several control charts that can detect small process mean shifts have been proposed, which have studied the properties of the weighting of the data or data depending on the period. Several researchers proposed a control chart that prioritizes historical data by qualifying the chart so that deficiencies from standard control charts can be corrected. The emphasis was on weighted historical data, such as the cumulative sum control chart (CUSUM) that was firstly proposed by Page [10]. Later in 1959, Roberts [11] presented an exponentially weighted moving average (EWMA) with a focus on historical data with exponential weighting down. Khoo [12] introduced the moving average (MA) chart, a control chart that calculates moving averages with the span (w) to adjust them smoothly. The results showed that when the is increased, it is good at detecting small changes. In 2008, Khoo and Wong [13] jointly developed the double moving average (DMA) chart, a control chart that takes the MA statistics to find one more time of moving average with the same value of span (w). The ability of MA and DMA charts are excellent to monitor small to moderate process changes mean can be applied to data with continuous and discrete distributions. Patel and Divecha [14] proposed a modified exponentially weighted moving average (MEWMA) control chart whose performance is superior to the traditional EWMA control chart for small changes. Recently, Sukparungsee *et al.* [15] proposed a new mixed exponentially weighted moving average – moving average (EWMA-MA) chart to detect a change in the process mean and Saengsura *et al.* [16] also presented a mixed moving average-cumulative sum (MA-CUSUM) chart to monitor parameter change.

However, there are many real word practices, which observations come from non-normal distributions or may not know the distribution of the process. Consequently, the traditional parameter-based control charts may lead to erroneous and impractical conclusions due to inconsistency in the preliminary agreement. To overcome these given limitations, the nonparametric control chart is a practicable, alternative to addressing this problem, i.e., using the Sign or Wilcoxon sign-rank (Sign-Rank) statistics applied to basic control charts that are effective in detecting changes, such as the EWMA, CUSUM, MEWMA charts. Yang *et al.* [17] presented a new EWMA Sign control chart and also recommended an Arc-Sign EWMA control chart which both charts are nonparametric control charts. In 1991, Amin and Searcy [18] established a distribution-free EWMA control chart based on the Wilcoxon signed-rank statistic confirming the superiority of their nonparametric scheme against the traditional $\overline{X}$ EWMA scheme under heavy-tailed distributions. Recently, Aslam *et al.* [19] proposed a control chart modified exponentially weighted moving average (MEWMA) chart by applying the sign statistic together named "Modified exponentially weighted moving average – Sign control chart (MEWMA-Sign)" chart using the average run length as a benchmark for measuring performance. The results showed that the MEWMA–Sign chart was more effective at detecting changes than the EWMA-Sign chart and EWMA chart, but was unable to detect process changes in the case of right-skewed distributions [20]. In 2020, Raza *et al.* [21] developed a nonparametric control chart so-called Homogeneously weighted moving average-Sign rank (NPHWMA-SN) for independent distributions and compared the performance to the NPEWMA-SN and NPCUSUM control charts. The performance of the NPHWMA-SN control chart showed superior detection performance than NPEWMA-SN and NPCUSUM for all changes.

Therefore, a new control chart is developed to enhance the detection performance of the control chart. This chart is a nonparametric control chart based on Wilcoxon sign rank statistic applying with the MEWMA chart, namely the MEWMA-Sign Rank (MEWMA-SR) chart, which combines the advantages of both charts. The MEWMA-SR can solve problems with processes that do not know the distribution of observations or are unable to estimate parameters where the general parametric control chart cannot resolve the constraints on this issue. The presented control charts have a better to moderate change detection performance than EWMA-Sign control charts and other control charts.

2 Control Charts and Properties

This section describes the conceptual design of the traditional parametric control charts as exponentially weighted moving average (EWMA) and modified exponentially weighted moving average (MEWMA) control charts and existing nonparametric control charts as Exponentially weighted moving average-Sign (EWMA-Sign), modified exponentially weighted moving average sign (MEWMA-Sign), Exponentially weighted moving average-Sign rank (EMWA-SR) [17] and the proposed modified Exponentially weighted moving average - Sign rank (MEWMA-SR) control charts.

Without loss of generality, we assumed that $\{X_{it}, 1 \le i \le n\}$ is an independent and identically distributed (i.i.d.) sequence of observations drawn from X at time t, which follows an arbitrary distribution. Generally, the function of the control chart is to detect the process mean deviation from the target value T, given $Y_{it} = X - T$ be the difference between the observation and the target value Y_{it}. Then, the value could be negative (including zero) or positive. Let p be a process proportion given p_0 is proportion when the process is in control; $p_0 = 0.5$ and p_1 is proportion when the process is out of control; $p_1 \ne 0.50$. The statistic Y_{it} can define as following

$Y_{it} = X_{it} - T$, for $i = 1, 2, ..., n$ and $t = 1, 2, ...$

and dummy variable $I_{it} = \begin{cases} 1, & \text{if } Y_{it} > 0 \\ 0, & \text{otherwise} \end{cases}$.

Therefore, the sign statistic $S_t = \sum_{i=1}^{n} I_{it}$ is the total number of observations $Y_{it} > 0$, that follows the binomial distribution with parameter (n, $p_0 = 0.5$) for in control state.

Let R_{it}^{+} denote the rank of the absolute difference $\left|X_{it} - T\right|$, $t = 1,2,...$ within the i^{th} subgroup. The sign rank statistics define as follows:

$$SR_t = \sum_{i=1}^{n} I_{it} R_{it}^{+},\ t = 1,2,...$$

where $I_{it} = \begin{cases} 1, & \text{when } \left(X_{it} - T\right) > 0 \\ 0, & \text{when } \left(X_{it} - T\right) = 0 \\ -1, & \text{when } \left(X_{it} - T\right) < 0 \end{cases}$.

2.1 *Exponentially weighted moving average (EWMA) control chart*

This control chart is typically time-weighted, which considering the historical data was proposed in 1959 by Roberts [11]. The ability of changed detections is excellent when a small change in the process means is detected. The EWMA statistics can describe as following Equation (1):

$$Z_t = \omega \bar{X}_t + \left(1 - \omega\right) Z_{t-1},\ t = 1,2,... \tag{1}$$

where ω is the weighted parameter of the data in the past having the values from 0 to 1, and $\bar{X}_t$ in the mean process at time t. The initial value of Z_0 is usually given to equal μ_0 where $\bar{X}_t$ (t = 1,2,...) are the independent and normally distributed observations, then the mean and variance of Z_t are

$$E\left(Z_t\right) = \mu_0$$

and

$$Var\left(Z_t\right) = \sigma^2 \left(\frac{\omega}{2-\omega}\left(1 - \left(1-\omega\right)^{2t}\right)\right),\ t = 1,2,... \tag{2}$$

From Equation (2), when $t \to \infty$, the asymptotic variance is

$$Var\left(Z\right) = \sigma^2 \left(\frac{\omega}{2-\omega}\right).$$

Therefore, the control limits of the EWMA control chart are the following Equation (3)

$$UCL_z / LCL_z = \mu_0 \pm \kappa_1 \sqrt{\sigma^2 \left(\frac{\omega}{2-\omega}\right)} \tag{3}$$

where κ_1 is a coefficient of the control limit of the EWMA control chart that corresponds with the desired ARL_0. This value can be determined by Monte Carlo simulation to correspond to the desired ARL_0. μ_0 is the mean of the process and the variance is σ^2.

2.2 *Exponentially weighted moving average-Sign (EWMA-Sign) control chart*

The mixed EWMA-Sign control chart is joined with the parametric EWMA chart based on the Sign statistic was presented by Yang *et al.* [17]. The EWMA-Sign statistic as Equation (4) can be written as follows:

$$Z_{S_t} = \omega S_t + (1-\omega) Z_{S_{t-1}}, \ t = 1, \ 2, ... \quad (4)$$

where ω is the weight constant, $0 < \omega \leq 1$. The mean and variance of Z_{S_t} for in control process are following:

$$E(Z_{S_t}) = np_0$$

and

$$Var(Z_{S_t}) = \frac{\omega\left(1-(1-\omega)^{2t}\right)\left(np_0(1-p_0)\right)}{2-\omega}. \quad (5)$$

When $t \rightarrow \infty$, Equation (5) can be rewritten as $Var(Z_S) = \frac{\omega\left(np_0(1-p_0)\right)}{2-\omega}$. The asymptotic upper and lower control limits for the EWMA-Sign control chart are presented by Equation (6) as follows:

$$UCL_{Z_S} / LCL_{Z_S} = np_0 \pm \kappa_2 \sqrt{\frac{\omega}{2-\omega}\left(np_0(1-p_0)\right)}, \quad (6)$$

where κ_2 is a coefficient of control limit of the EWMA-Sign control chart that corresponds with the desired ARL_0. The EWMA-Sign statistics will signal to be out of control process when $Z_{S_t} > UCL_{Z_S}$ or $Z_{S_t} < LCL_{Z_S}$.

2.3 *Exponentially weighted moving average-Sign rank (EWMA-SR) control chart*

This control chart was introduced by Yang *et al.* [17] which is popular in the production process to have a normal distribution. However, it will be found the production process may also have non-normal distributions. For this reason, a nonparametric control chart is introduced, namely the EWMA-SR control chart, which is a control chart used to detect changes in the mean process. The EWMA-SR statistic can define in a mathematical term as Equation (7)

$$Z_{SR_t} = \omega SR_t + (1-\omega) Z_{SR_{t-1}}, \ t = 1, \ 2, ... \quad (7)$$

where ω is the weight constant, $0 < \omega \leq 1$. The mean and asymptotic variance of Z_{SR_t} for in control process is following:

$$E(Z_{SR}) = 0$$

and

$$Var(Z_{SR}) = \frac{\omega}{2-\omega}\left(\frac{n(n+1)(2n+1)}{6}\right).$$

Therefore, the upper and lower control limits of EWMA-SR are following

$$UCL_{Z_{SR}} / LCL_{Z_{SR}} = \pm\kappa_3 \sqrt{\frac{\omega}{2-\omega}\left(\frac{n(n+1)(2n+1)}{6}\right)}$$

where κ_3 is a coefficient of control limit of the EWMA-SR control chart that corresponds with the given ARL_0. The EWMA-SR will notice to be out of control when $Z_{SR_t} > UCL_{Z_{SR}}$ or $Z_{SR_t} < LCL_{Z_{SR}}$.

2.4 *Modified exponentially weighted moving average (MEWMA) control chart*

This control chart was developed to enhance the detection performance by adding the last term $k(S_t - S_{t-1})$ to the EWMA statistic which made the MEWMA-Sign statistics are higher than the EWMA-Sign control chart for the same data sets. The difference between the EWMA and the MEWMA Charts which the latter adds $k(S_t - S_{t-1})$ term to the EWMA statistic. The performance of the charts is compared using the observations in the analysis. The study found that the MEWMA chart was able to detect changes better than the EWMA chart at all levels of change.

$$MZ_t = (1-\omega)MZ_{t-1} + \omega X_t + k(X_t - X_{t-1}); \ t = 0,1,2,...$$

where k is a constant. If the value of k is given to 1. The MEWMA-SR statistic is in the same manner of the form of Patel and Divecha [14] when given $k = 1$. The mean and variance of MZ_t for in control process are following:

$$E(MZ) = \mu_0$$

and

$$Var(MZ_t) = \sigma^2\left(\frac{\left(\omega + 2\omega k + 2k^2\right) - \omega(1-\omega-k)^2(1-\omega)^{2(t-1)}}{2-\omega}\right).$$

The upper control limit and lower control limit of the MEWMA Chart when $t \to \infty$ are defined as follows:

$$UCL_{MZ} / LCL_{MZ} = \mu_0 \pm \kappa_4 \sigma \sqrt{\frac{\omega + 2\omega k + 2k^2}{2-\omega}}$$

where κ_4 is a control limit coefficient of the MEWMA that corresponds with the given ARL_0.

2.5 *Modified exponentially weighted moving average-Sign (MEWMA-Sign) control chart*

Aslam *et al.* [19] presented the MEWMA-Sign control chart which compared the detection performance with other control charts. The MEWMA-Sign statistic describes as Equation (8)

$$MZ_{S_t} = \omega S_t + (1-\omega) MZ_{S_{t-1}} + k\left(S_t - S_{t-1}\right),\ t = 1,\ 2,... \quad (8)$$

where k is a constant and given equal to 1 and it coincides with the form of Patel and Divecha [14] when set up $k = 1$. Furthermore, Equation (8) coincides with the EWMA-Sign control chart when given $k = 0$. The mean and variance of MZ_{S_t} are as follows:

$$E\left(MZ_{S_t}\right) = np_0$$

and

$$Var\left(MZ_{S_t}\right) = \left[\frac{\left(\omega + 2\omega k + 2k^2\right) - \omega\left(1-\omega-k\right)^2\left(1-\omega\right)^{2(t-1)}}{2-\omega}\right] np_0\left(1-p_0\right). \quad (9)$$

From Equation (9), when $t \to \infty$, then

$$Var\left(MZ_S\right) = \left(\frac{\omega + 2\omega k + 2k^2}{2-\omega}\right) np_0\left(1-p_0\right).$$

Therefore, the asymptotic upper and lower control limits for the MEWMA-Sign control chart are given by Equation (10) as follows:

$$UCL_{MZ_S} / LCL_{MZ_S} = np_0 \pm \kappa_5 \sqrt{\frac{\omega + 2\omega k + 2k^2}{2-\omega}\left(np_0\left(1-p_0\right)\right)}, \quad (10)$$

where κ_5 is a control limit coefficient of the MEWMA-Sign control chart that corresponds with the given ARL_0.

2.6 *Modified exponentially weighted moving average-sign rank (MEWMA-SR) control chart*

MEWMA-SR control chart evolved from MEWMA Sign with an additional rank term when the process has a small change. Using the SR statistic based can be better used to check the median of the process. The statistical value of the MEWMA-SR chart is

$$MZ_{SR_t} = \omega SR_t + (1-\omega) MZ_{SR_{t-1}} + k(SR_t - SR_{t-1}).$$

The mean and asymptotic variance of MZ_{SR_t} for in control process is following:

$$E\left(MZ_{SR}\right) = 0$$

and

$$Var\left(MZ_{SR}\right) = \frac{(\omega + 2\omega k + 2k^2)}{(2-\omega)}\left(\frac{n\left(n+1\right)\left(2n+1\right)}{6}\right).$$

The asymptotic control limits of the MEWMA-SR control chart are as follows.

$$UCL_{MZ_{SR}} / LCL_{MZ_{SR}} = \pm \kappa_6 \sqrt{\frac{(\omega + 2\omega k + 2k^2)}{(2-\omega)}\left(\frac{n(n+1)(2n+1)}{6}\right)}$$

where κ_6 is a control limit coefficient of the MEWMA-SR control chart that corresponds with the given ARL_0.

3 Performance Measures and Properties

The run length (RL) is a random variable of the size of samples until the warning signal is detected by the control chart. If the processes are performing and the warning signal is detected, this error detection is a Type I error. One of the measurement methods for detecting a change in the process is the average run length (ARL), which is a popular criterion used to evaluate the efficiency of a control chart. The ARL value measures the efficiency of the control chart about detecting the amount of waste in the production process, which is determined by the quickness of the detection of the observed value outside the control when the mean process has changed. The control chart that can detect the change quickly is considered efficient because the quick detection allows for the causes and solutions to be identified immediately. The ARL is the average number of points that must be plotted before a point indicates an out-of-control condition when ARL_0 is in an in-control state and ARL_1 is in an out-of-control state. ARL_0 is defined as the measure of time before a process that is on target when a process is falsely signaled as being out-of-control, and then is defined as:

$$ARL_0 = E_\theta(\tau) = C, \ \theta = \infty$$

where $E_\theta(\tau)$ is the expectation of the stopping time
τ is a stopping time
θ is a change-point time
C is a constant (should be large enough).

Otherwise, when the process is out of control, the ARL_1 describes the detection performance. ARL_1 is implemented as a measure of the time before the process that has gone out of control when a process is signaled as being out of control. Ideally, ARL_1 must be minimum, which can be written as:

$$ARL_1 = E_\theta\left(\tau - \theta + 1 \middle| \tau \geq \theta\right), \quad (11)$$

where $E_\theta(\tau)$ is the expectation of an assumption that a change-point has occurred in time θ. Note that, Equation (11) is usually determined when a change occurs at the beginning process $\theta = 1$. In this research, the approximated ARL_0 and ARL_1 are carried out by using the Monte Carlo simulation with $m = 10^5$ replications as Equation (12):

$$ARL = \frac{\sum_{j=1}^{m} RL_j}{m}. \quad (12)$$

The ARL has explained the RL properties of all investigated control charts; EWMA, MEWMA, EWMA-Sign, MEWMA-Sign, EWMA-SR, and MEWMA-SR control charts. The numerical results are presented in Tables 1–6 by varying the rational subgroup (n) = 5, 10, and 15 and magnitudes of shifts in mean parameter.

4 Simulation Results and Real Applications

In this research, we proposed a new MEWMA based Sign Rank; MWMA-SR, and also compared the detection efficiency with other existing control charts. The numerical results are carried out from the Monte Carlo simulation using R programming software [22] as shown in the value of ARL_0 and ARL_1. In addition, the real word is investigated to show the performance in detecting a change in the mean parameter using the real monthly S&P 500 index data set. However, the proposed control chart can be arbitrary distributions, then in this paper study on the Lognormal(0,1) and Laplace(1,1) distributions which usually occurred in industrial problems. The MEWM-SR control chart has compared the performance of change detection with parametric control charts as EWMA and MEWMA control charts and nonparametric control charts based on the Sign and Sign Rank statistics as EWMA-Sign, MEWMA-Sign, MEWMA-SR control charts by considering the best performance from the minimum value of ARL_1. In addition, the rational subgroup (n) is also studied by categorized to be 3 sizes as n = 5, 10, 15 and given the smoothing parameter of exponential based control chart where ω = 0.1 for Lognormal distribution and Laplace distribution. This smoothing parameter is usually chosen as a small value because they are excellent to detect a small change. The six coefficients of control limits for each control chart κ_j; j = 1, 2,..., 6 are determined with the Monte Carlo simulations to correspond with the desired ARL_0 = 370. Tables 1–3 show the results of ARL_0 and ARL_1 when observations are from Lognormal distribution with in-control parameter and rational subgroup are 5, 10, and 15, respectively. The out-of-control parameter of the process distribution, α_1 is given to $\alpha_0(1 + \delta)$, where δ is a value of magnitude of changes was varied from 0.02, 0.05, 0.1, 0.15, 0.2, 0.3, 0.5, 0.7, and 1.0. In addition, the numerical results are from Laplace distribution with in-control parameter shown on Tables 4–6 when rational subgroup are 5, 10, and 15, respectively. The out-of-control parameter of the process distribution, is given to where is a value of magnitude of changes was varied from 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.5, 1.0, 1.5, and 2.0.

Tables 1–3 present the numerical results for various rational subgroups n = 5, 10, and 15, respectively when observations have Lognormal distribution and given ARL_0 = 370. We found that the proposed control charts are superior to other existing control charts when shift sizes (δ) are small to moderate; $\delta > 0.05$ for n = 5, otherwise the EWMA-SR shows better performance when small changes $\delta < 0.10$. For n = 10, the proposed control chart shows in the same manner with n = 5, the MEWMA-SR is excellent to detect small to moderate changes $\delta > 0.02$, otherwise, the MEWMA and MEWMA-Sign will perform similarly when $\delta > 0.30$ as clearly graphical viewed as Figure 1. In addition, the MEWMA-SR outperforms in the

Table 1: Comparison of the chart's performance with ω = 0.10 and n = 5 for Lognormal distribution

δ	Control Charts					
	EWMA	EWMA-Sign	EWMA-SR	MEWMA	MEWMA-Sign	MEWMA-SR
	K_1 = 17.443	K_2 = 8.721	K_3 = 17.498	K_4 = 5.635	K_5 = 2.817	K_6 = 3.514
0	370.112	370.442	370.989	370.3401	370.564	370.248
0.02	311.468	313.578	262.564*	355.330	354.669	280.457
0.05	247.306	239.475	167.554*	289.800	289.065	179.565
0.10	159.435	158.273	94.541	168.568	168.238	91.914*
0.15	114.28	111.953	59.996	96.871	96.726	54.168*
0.20	89.298	89.89	44.142	60.963	60.914	36.144*
0.30	60.175	59.582	27.11	31.233	31.178	20.284*
0.50	41.087	40.508	16.586	14.3106	14.306	10.507*
0.70	33.345	33.172	12.382	9.207	9.195	7.254*
1.00	28.007	28.268	9.557	6.183	6.179	5.223*

Note: * is the minimum value of ARL_1

Table 2: Comparison of the chart's performance with ω = 0.10 and n = 10 for Lognormal distribution

δ	Control Charts					
	EWMA	EWMA-Sign	EWMA-SR	MEWMA	MEWMA-Sign	MEWMA-SR
	K_1 = 23.599	K_2 = 8.967	K_3 = 18.124	K_4 = 5.699	K_5 = 2.849	K_6 = 4.346
0	370.627	370.679	370.985	370.127	370.623	370.637
0.02	272.347	195.258*	306.625	338.642	338.505	246.531
0.05	193.287	154.438	204.258	232.440	231.885	133.929*
0.10	117.391	120.744	120.531	101.913	101.463	60.629*
0.15	81.22	92.8641	82.073	51.740	51.769	35.267*
0.20	60.478	57.654	63.593	31.261	31.155	24.081*
0.30	45.002	33.770	45.103	15.677	15.642	14.541*
0.50	30.978	17.054	31.69	7.343*	7.345*	8.397
0.70	26.319	11.5778	26.745	4.784*	4.788*	6.178
1.00	22.991	7.460	23.059	3.201*	3.198*	4.690

Note: * is the minimum value of ARL_1

Table 3: Comparison of the chart's performance with ω = 0.10 and n = 15 for Lognormal distribution

δ	Control Charts					
	EWMA	EWMA-Sign	EWMA-SR	MEWMA	MEWMA-Sign	MEWMA-SR
	K_1 = 28.301	K_2 = 9.328	K_3 = 19.697	K_4 = 5.73	K_5 = 2.8612	K_6 = 4.958
0	370.833	370.435	370.624	370.686	370.261	370.215
0.02	308.373	293.911	215.507*	327.448	322.733	225.9228
0.05	221.451	253.097	112.516	191.959	190.855	109.841*
0.10	124.128	185.832	57.306	71.092	70.393	47.612*
0.15	80.787	159.348	37.306	34.225	34.162	28.150*
0.20	57.278	121.843	28.657	20.353	20.264	19.407*
0.30	37.289	67.893	20.175	10.292	10.265*	12.440
0.50	23.053	34.282	13.56	4.847*	4.837*	7.522
0.70	18.698	14.393	10.678	3.114*	3.116*	5.659
1.00	17.077	11.935	8.634	2.029*	2.031*	4.359

Note: * is the minimum value of ARL_1

same manner with n = 10. We also investigate the performance of the MEWMA-SR with symmetric distribution as Laplace(1,1) distribution under various rational subgroup n = 5, 10, and 15 as shown in Tables 4–6, respectively. The numerical results found that the proposed control chart, MEMWA-SR is outperformance when implemented with small subgroup n = 5 and large changes $\delta > 1.0$. Otherwise, the EWMA-SR is superior to other control charts for small changes as clearly graphical shown in Figure 2.

Table 4: Comparison of the chart's performance with ω = 0.05 and n = 5 for Laplace distribution

δ	Control Charts					
	EWMA	**EWMA Sign**	**EWMA-SR**	**MEWMA**	**MEWMA Sign**	**MEWMA-SR**
	K_1 **= 0.746**	K_2 **= 2.708**	K_3 **= 9.261**	K_4 **= 0.719**	K_5 **= 2.454**	K_6 **= 1.053**
0	370.099	370.267	370.210	370.455	370.947	370.210
0.05	356.754	306.742	220.934*	365.020	366.091	365.241
0.10	307.923	214.790	136.162*	350.555	350.709	310.638
0.15	249.756	142.173	89.346*	321.044	347.091	282.752
0.20	194.997	95.972	61.171*	300.555	335.681	242.788
0.30	144.182	67.007	43.819*	279.331	332.478	210.009
0.50	112.858	48.506	32.790*	190.998	329.691	149.894
1.00	40.381	18.486	14.081*	53.138	287.831	10.471
1.00	6.807	5.761	5.512*	10.096	182.923	81.839
1.50	2.842	3.557	3.937	1.851	87.208	0.595*
2.00	1.944	2.800	3.395	0.818	46.785	0.326*

Note: * is the minimum value of ARL_1

Table 5: Comparison of the chart's performance with ω = 0.10 and n = 10 for Laplace distribution

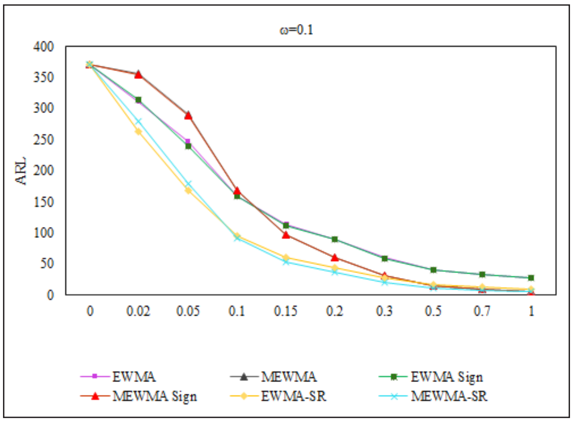
δ	Control Charts					
	EWMA	**EWMA Sign**	**EWMA-SR**	**MEWMA**	**MEWMA Sign**	**MEWMA-SR**
	K_1 **= 15.712**	K_2 **= 2.716**	K_3 **= 69.710**	K_4 **= 15.251**	K_5 **= 2.831**	K_6 **= 42.156**
0	370.426	370.014	370.433	370.236	370.520	370.714
0.05	298.349	291.322	182.021*	332.214	356.274	257.384
0.10	204.927	153.468	101.341*	276.255	271.370	214.349
0.15	163.827	94.439	64.325*	231.315	211.238	183.279
0.20	74.214	55.238	31.280*	159.350	169.214	123.347
0.30	57.2846	21.328	20.324*	91.233	124.217	107.496
0.50	21.341	17.214	16.325*	77.189	86.026	81.239
1.00	14.699	10.214*	9.216	58.262	32.382	43.241
1.00	6.294	3.672*	6.321	32.361	12.249	12.340
1.50	1.974	1.275	2.838	25.180	0.422*	3.496
2.00	0.254	0.725	1.391	14.438	0.002*	0.349

Note: * is the minimum value of ARL_1

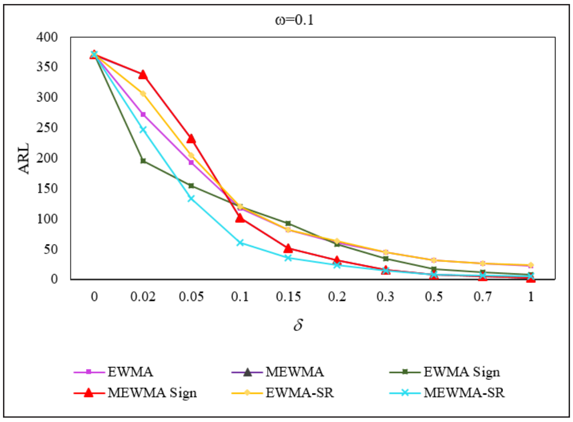
Table 6: Comparison of the chart's performance with ω = 0.10 and n = 15 for Laplace distribution

δ	Control Charts					
	EWMA	**EWMA Sign**	**EWMA-SR**	**MEWMA**	**MEWMA Sign**	**MEWMA-SR**
	K_1 **= 22.05**	K_2 **= 2.942**	K_3 **= 71.124**	K_4 **= 21.916**	K_5 **= 3.09**	K_6 **= 42.723**
0	370.034	370.117	370.677	370.338	370.844	370.385
0.05	259.572	260.691	165.453*	361.049	330.694	269.656
0.10	127.152	128.020	81.444*	331.514	263.066	197.502
0.15	63.998	64.523	44.835*	288.352	195.499	146.790
0.20	35.601	35.720	27.587*	238.401	142.207	110.207
0.30	21.709	21.834	18.509*	192.427	100.641	82.471
0.50	14.374	14.469	13.426*	157.870	71.146	61.944
1.00	4.699	4.744*	6.141	90.164	14.850	20.001
1.00	1.417	1.416	3.212	34.5342	0.115*	2.355
1.50	0.895	0.893	2.577	30.820	0.002*	1.149
2.00	0.657	0.656	2.227	19.985	0.000*	1.040

Note: * is the minimum value of ARL_1



(a)



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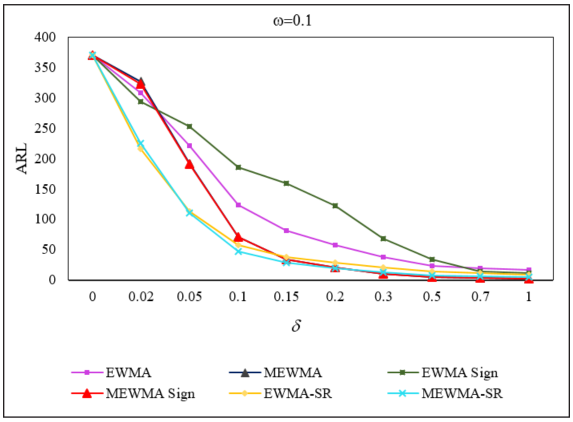
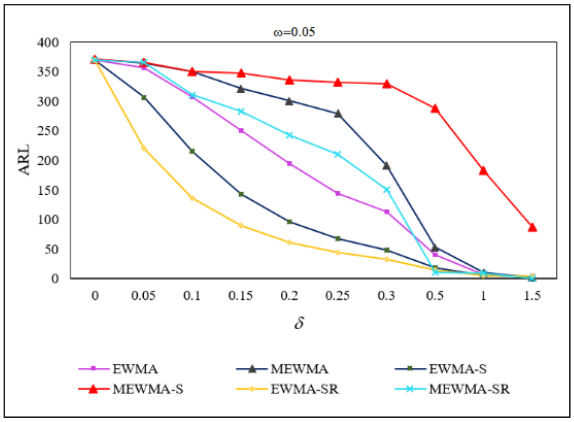
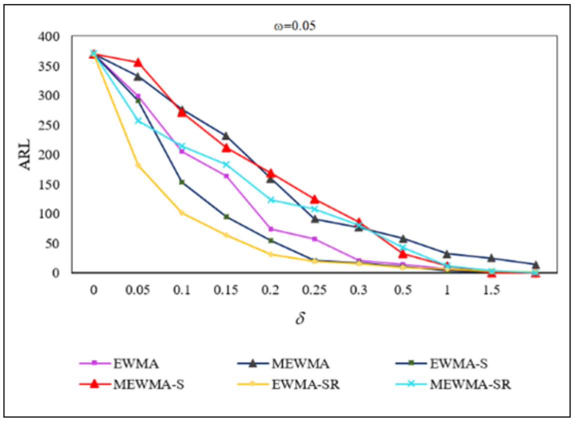


(c)

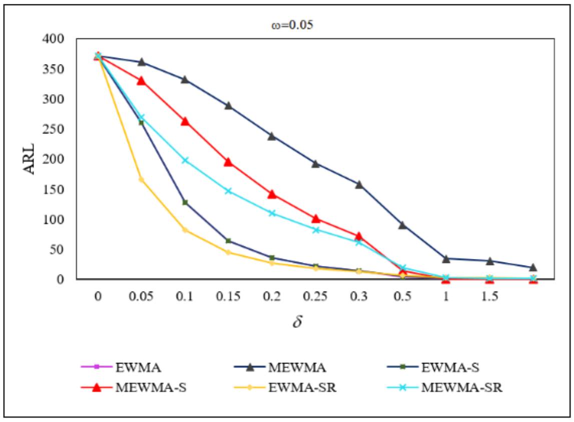
Figure 1: ARL curves of EWMA, EWMA-Sign, EWMA-SR, MEWMA, MEWMA-Sign and MEWMA-SR control charts for Lognormal distribution (a) n = 5, (b) n = 10, and (c) n = 15.



(a)



(b)



(c)

Figure 2: ARL curves of EWMA, EWMA-Sign, EWMA-SR, MEWMA, MEWMA-Sign and MEWMA-SR control charts for Laplace distribution (a) n = 5, (b) n = 10, and (c) n = 15.

For the real world application, we have selected real monthly S&P 500 index data between June 2015 to May 2018 under Lognormal distribution with scale parameter α = 2281.55, shape parameter β = 0.113, and the significance to be Lognormal distribution is presented with p-value = 0.077 and P-P plot as Figure 3. The data set are totally 36 observations 2705.27, 2648.05, 2640.87, 2713.83, 2823.81, 2673.61, 2647.58, 2575.26, 2519.36, 2471.65, 2470.3, 2423.41, 2411.8, 2384.2, 2362.72, 2363.64, 2278.87, 2238.83, 2198.81, 2126.15, 2168.27, 2170.95, 2173.6, 2098.86, 2096.96, 2065.3, 2059.74, 1932.23, 1940.24, 2043.94, 2080.41, 2079.36, 1920.03, 1972.18, 2103.84, 2063.11. The performance of detecting a change in a mean parameter of EWMA, EWMA-Sign, EWMA-SR, MEWMA, MEWMA-Sign, and the MEWMA-SR are presented by showing as graphical displays in Figure 4(a)–(f), respectively. The graphical results show that the proposed MEWMA-SR chart can signal at the first time to be out of control on sample 28th. Otherwise, the other control charts could not detect any change. Therefore, the performance of the MEWMA-SR chart is the best performance to detect a change in the S&P 500 index when compared with the existing control chart.

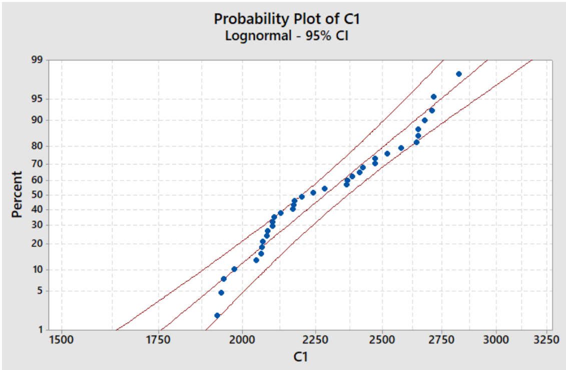
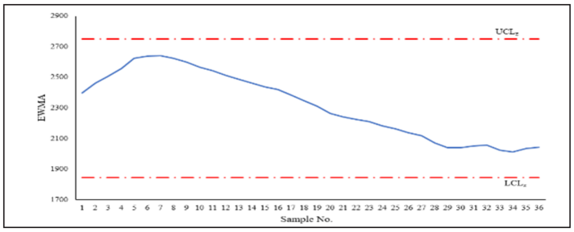
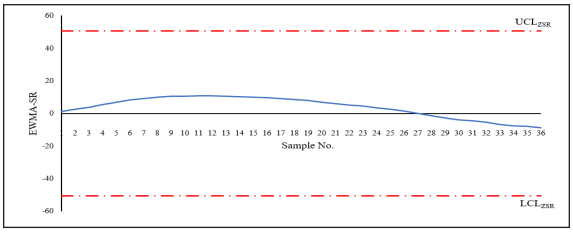


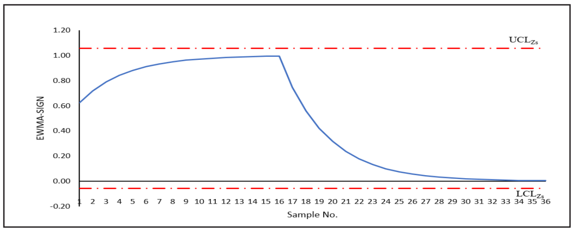
Figure 3: P-P plot for Lognormal distribution of S&P index data set.



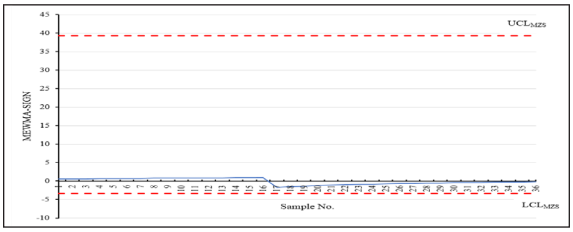
(a) EWMA



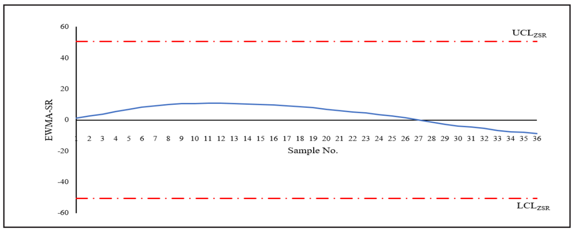
(d) EWMA-SR



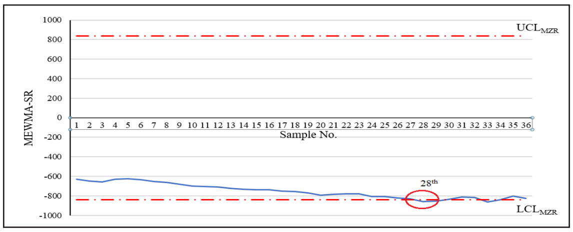
(b) EWMA-Sign



(e) MEWMA-Sign



(c) EWMA-SR



(f) MEWMA-SR

Figure 4: Comparison of detection performance of the S&P 500 index during 2015–2018 between existing charts (a) EWMA, (b) EWMA-Sign, (c) EWMA-SR, (d) MEWMW, (e) MEWMA-Sign, and (f) MEWMA-SR.

5 Discussion and Conclusions

The objective of this research is to propose a new mixed parametric control chart as MEWMA with nonparametric statistics as Sign Rank, the so-called MEWMA-SR control chart. This proposed chart can use when observations are the arbitrary or unknown distribution that overcomes the limitation of traditional parametric control chart and is very useful to many real situations. The performance of the proposed MEWMA-SR control chart is compared with EMWA, EWMA-Sign, EWMA-SR, MEWMA, and MEWMA-Sign control charts to detect a change in the process mean parameter. The assessments and numerical results prove that the MEMWA-SR control chart outperforms the EMWA, EWMA-Sign, EWMA-SR, MEWMA, and MEWMA-Sign control charts for the case of Lognormal distribution, when a magnitude of change is a small change, and rational subgroup is small. Otherwise, the performance of nonparametric control charts as such EWMA-Sign, EWMA-SR, MEWMA-Sign control charts are better than parametric control charts, which the former can overcome the limitation of assumption of process observation that must be known parameter or the parameter of distribution can be estimated. The performance of the MEWMA chart combined with either the Sign and Sign Rank statistics show the better performance of EWMA based control chart for Lognormal distribution for except the case of and n = 5, the EWMA-SR is superior to other control charts. For the Laplace distribution, the performance of EWMA-SR and MEWMA-SR is no different when a magnitude of change is large Otherwise, the EWMA based on Sign and Sign Rank statistics performs better than the proposed control chart. Consequently, the choice of optimal control chart for lognormal and Laplace distributions for each rational subgroup and various magnitudes of change are addressed in Table 7.

Furthermore, there is an effective alternative to nonparametric statistics, which enhanced the performance of detection of a change for two independent variables such as Mann-Whitney statistic, which can implement in bivariate and multivariate nonparametric control charts. The optimal of value of the MEWMA-Sign or MEWMA-SR control charts could not be theoretically derived, however, the author will study the effect of values to raise the performance of detection in further research.

Table 7: Choices of optimal control charts for detection of a change in the process mean

Dist.	δ	Rational Subgroup (n)		
		5	10	15
Log-normal	$0.02 \leq \delta \leq 0.05$	EWMA-SR	MEWMA-SR	MEWMA-SR
	$0.1 \leq \delta \leq 0.3$	MEWMA-SR	MEWMA-SR	MEWMA-SR
	$0.5 \leq \delta \leq 1.0$	MEWMA-SR	MEWMA, MEWMA-Sign	MEWMA, MEWMA-Sign
Laplace	$0.05 \leq \delta \leq 0.15$	EWMA-SR	EWMA-SR	EWMA-SR
	$0.15 \leq \delta \leq 0.25$	EWMA-SR	EWMA-SR	EWMA-SR
	$0.25 \leq \delta \leq 0.50$	EWMA-SR	EWMA-Sign, EWMA-SR	EWMA-Sign, EWMA-SR
	$0.50 \leq \delta \leq 2.0$	EWMA-SR, MEWMA-SR	EWMA-Sign, MEWMA-Sign	EWMA-Sign, MEWMA-Sign

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