Gear Tooth Profile Error caused by Hob Misalignment

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Abstract

By a relatively simple method to estimate the tooth profile errors caused by misalignment of the hob, the relation between the tooth profile error of the gear and any type of radial run-out of the hob are calculated. The involute curve test chart waves usually under the influence of radial run-out of the hob but the almost true involute tooth profile is generated under a special run-out condition. The software made from BASIC language is developed to show hobbing machine operators the importance of hob tooling.

Keywords*: Involute gear, Hobbing, Tooth profile error*

1 Introduction

The cutting teeth of a hob generate teeth surfaces of an imaginary rack during the rotation of the hob to mesh with a work gear. If the profile of the rack tooth would be insufficient, a good involute profile of the work gear could not be generated even if a highly accurate hobbing machine is used. The accuracy of the imaginary rack is determined also by the accuracy of the hob itself or the preciseness of hob mounting on the hob arbor. The accuracy of tooth form of the imaginary rack affects to a profile quality of work gear directly on the hobbing process. Authors developed the program which predicts the error under the condition that the maximum radial run-out of both side are on one plane and checked the simulation by experiment.[1-2]

In the following sections, improved program applicable to any run-out situation is introduced. Any run-out situation means different run-out value or different position of maximum run-out.

2 POSITION OF CUTTING EDGE

The teeth of a hob have to represent a helical rack moves in the direction of tangent to a pitch circle of a work gear with helix angle while the hob is rotated. On the tooling, it is necessary to incline the hob arbor axis at angle of the helix angle minus or plus the lead angle of the hob thread.

Figure 1 shows a helical gear hobbing condition. The hob mounted on the hob arbor is inclined at an angle to the work gear surface. In this report, we call a line perpendicular to both the hob arbor axis and the work gear axis a “generating center line”. As the hobbing motion is similar to a gearing of closed helical gear, the relative motion between the cutting teeth of the hob and the gear blank seems to be complex. But as both the parts are simply rotated around each axis, the deriving equations to estimate the tooth profile error are not difficult and those are provided as follows.

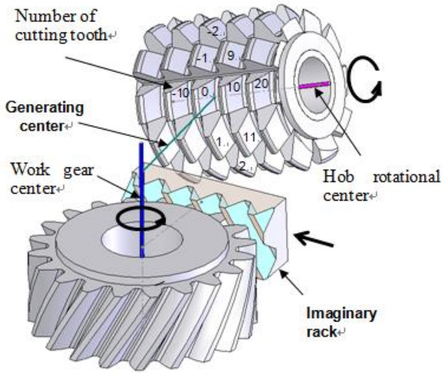


Figure 1: Imaginary rack, Generating center, and Number of cutting tooth

Figure 2 is projected pictures showing a movement of a point on a cutting edge of the hob. The picture (A) is the Horizontal view (Top view), the picture (B) is the Vertical view (Frontal view) from the hobbing machine column side, picture (C) is axial view, and the axial true form of the cutting tooth can be shown on picture (D). The point Pk is the intersection of a cutting a cutting edge and the line of action.

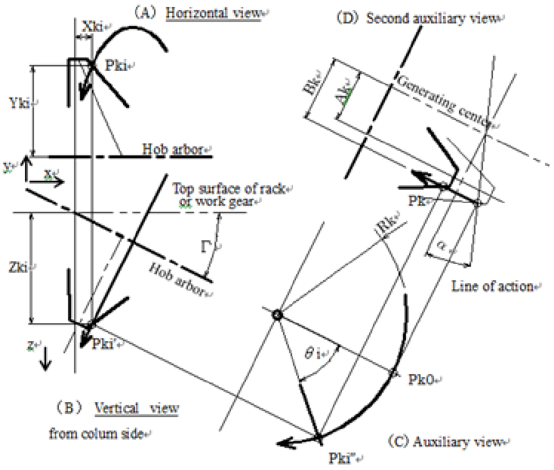


Figure 2: Movement of a generating point P_k

The point generates involute tooth profile. Here, the suffix k shows the number of cutting tooth as shown in Figure 1. The center of a cutting tooth, k = 0 is on the generating center. The point is simply rotated around the hob arbor center and the projection of the movement of the point is drawn with thick line or curve on each project plane in figure 2. When the cutting edge is rotated for an angle θ_i ,the position of P_{ki} (X_{ki} , Y_{ki}, Z_{ki}) is

$$X_{ki} = A_k \cdot \cos\Gamma - R_k \cdot \sin\theta_i \cdot \sin\Gamma$$
$$Y_{ki} = R_k \cdot \cos\theta_i \qquad (1)$$
$$Z_{ki} = A_k \cdot \sin\Gamma + R_k \cdot \sin\theta_i \cdot \cos\Gamma$$

Where A_k = Axial distance from generating center to the point P_k
Γ = Inclination angle of hob arbor
R_k = Radial distance from center of hob arbor to the point P_k

3 AMOUNT OF TOOTH PROFILE ERROR

The tooth profile of hobbed gear can be estimated by calculating the distance between the point Pki and the surface of imaginary rack. The regular axial tooth profiles of the rack are represented on the horizontal view (A). The positions can be decided as follows.

On the view (A) in figure 3, a right tooth flank E of imaginary rack is at the regular position on top surface of the rack when the rotational angle θ_i of No. k cutting tooth is 0 degree. The distance of the pitch point from the generating center is

$$X_{k0} = L_h \cdot \frac{k}{N} + \frac{S_h}{2} \qquad (2)$$

Where Lh = Axial pitch of rack
K = Cutting tooth number
N = Number of gash
Sh = Rack tooth thickness

After hob rotation, the rack moves from E to F along the hob pitch line. The distance Xt is

$$X_t = L_h \cdot \frac{\theta_i}{2\pi} \qquad (3)$$

At the time, the cutting point moves to a point P_{ki}’ on the front view. It is necessary to find the tooth profile of the rack on a horizontal plane including the point P_{ki}’. The axial distance ΔX_{ki} between the rack and the point is shown on the top view.

$$\Delta X_{ki} = X_{k0} - X_t + Z_{ki} \cdot \tan\beta - (Y_{ki} - R) \cdot \tan\alpha - X_{ki} \qquad (4)$$

Where β = Helix angle of imaginary rack
R = Radius of hob pitch cylinder
α = Pressure angle of the rack

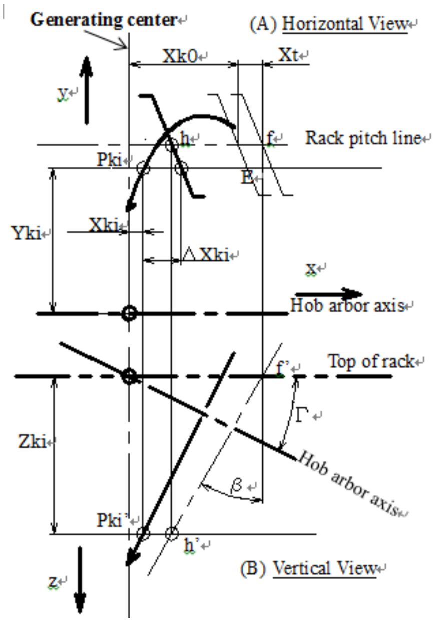


Figure 3: Distance between a generating point and a surface of imaginary rack

Finally, the shortest distance between the point Pki and the surface of imaginary rack is

$$\delta_{ki} = \Delta X_{ki} \cdot \cos\alpha \cdot \cos\beta \tag{5}$$

The distances δ_{ki} are calculated by dividing the rotation angle every 0.01 degree and the smallest value is assumed to tooth profile error of imaginary rack.

4 RADIAL RUN-OUT OF HOB

The radial run-outs are measured at both sides of hob as shown figure 4 using small test. In figure 5, Lm is a distance between both measuring points and Lg is a distance between generating center and the main bearing side measuring point.

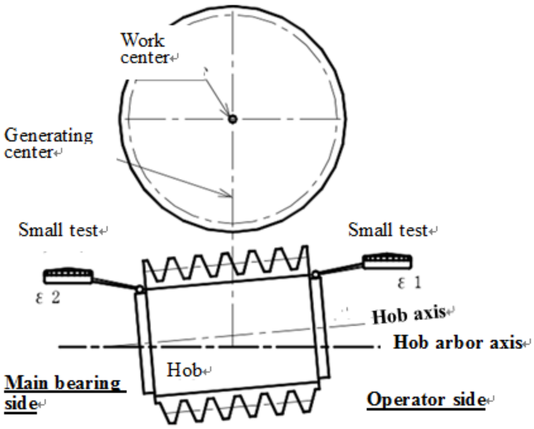


Figure 4: Measuring run-out of hob

The hob axis is inclined at the hob arbor axis by run-out as shown figure 6. In case that both maximum run-out points have shifted ϕ_{off}, the hob axis is shown as line M_1-N_2 and when the cutting tooth number k is in the position which angle ϕ_k rotated, the hob axis (line M'_1 - M'_2) is inclined at angle ζ_k and the radial run-out of each cutting tooth is changed gradually. The amount of the radial run-out ε_k of a cutting tooth, the cutting number is k, is expressed by

$$\zeta_k \cong \tan^{-1}\frac{\{\varepsilon_1 \cdot \cos\phi_k + \varepsilon_2 \cdot \cos(\phi_k + \phi_{off})\}}{L_m} \tag{6}$$

In the condition, following equations are set up to obtain the value of both Ak and Rk in (1)

On the top view, the center of a hob tooth moves to a point Ok and the coordinate is

$$\begin{aligned} X_k &= (L_g + L_x) \cdot (1 - \cos\zeta_k) \\ Y_k &= (L_g + L_x) \cdot \sin\zeta_k + \varepsilon_2 \cdot \cos(\phi_x + \phi_{off}) \end{aligned} \tag{7}$$

When the hob axis is inclined left at an angle ζ_k and the cutting tooth is shifted from K_1 to K_2, the coordinate of a point Pkp on the pitch line of the cutting tooth is

$$X_{kp} = X_k + R \cdot \sin\zeta_k \pm \frac{S_h}{2} \cdot \cos\zeta_k \cdot \cos\xi_k$$

$$Y_{kp} = Y_k + R \cdot \cos\zeta_k \mp \frac{S_h}{2} \cdot \sin\zeta_k \qquad (8)$$

Where ξ_k is an inclination angle of the cutting surface.

$$\xi_k \cong \tan^{-1}\frac{\varepsilon_1 \cdot \sin\phi_x - \varepsilon_2 \cdot \sin(\phi_x + \phi_{off})}{L_m \cdot \tan\zeta_k} \qquad (9)$$

A generating point Pkg is provided as an intersection of the cutting edge and the line of action of imaginary rack even when the cutting tooth is shifted and inclined. The coordinate of the point is

$$X_{kg} = \frac{(R - Y_{kp}) \cdot \tan(\alpha + \zeta_k) - X_{kp}}{1 - \tan\alpha \cdot \tan(\alpha + \zeta_k)}$$

$$Y_{kg} = \mathbf{R} - X_{kg} \cdot \tan\alpha \qquad (10)$$

Here, substituting Xkg into Ak and Ykg into Rk in (1), a tooth profile error can be approximately estimated by (2) to (8).

5 SIMULATION

It can be judged from no error happened in case of hobbing by involute hob under the condition of using perfect hob and machine that this program is able to use for estimation of the tooth profile error.[2]

Figure 7 (a) shows a simulated profile error of a gear cut by a standard hob with straight cutting edges when the hob is aligned and generated ideally.

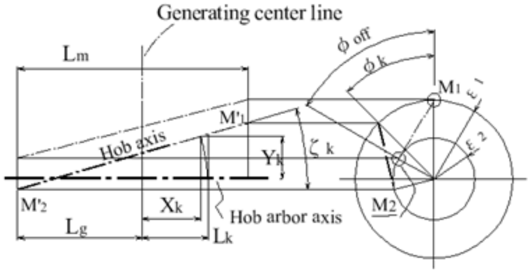


Figure 5: Inclination of hob axis in case of different run-out value and position

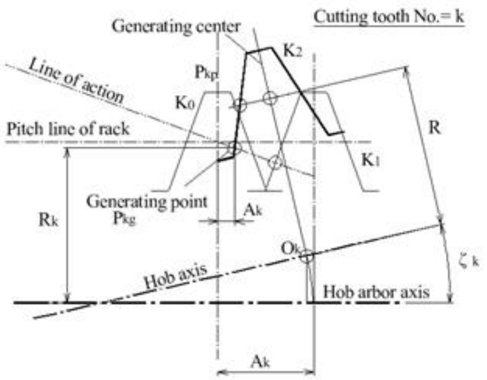


Figure 6: Generating point Pkg of a cutting tooth after inclination of hob axis

Table 1: Data of hob and gear for simulation

Hob

Normal module	5 mm
Normal pressure angle	20 degree
Outside diameter	95 mm
Width	90 mm
Number of thread start	1
Lead angle	3.51 degree
Number of Gashes	10
Accuracy	JIS AA

Gear

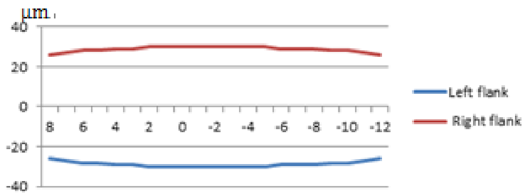
Normal module	5 mm
Transverse module	5.774 degree
Transverse pressure angle	22.796 degree
Number of teeth	20
Helix angle	30 degree
Pitch diameter	115.470 mm
Material	Brass bar

The size of the hob is a JIS standard hob shown in table 1 .The gear tooth is slightly cut off at both tip and root and the amounts of the relief close to theoretical value (4 μm).

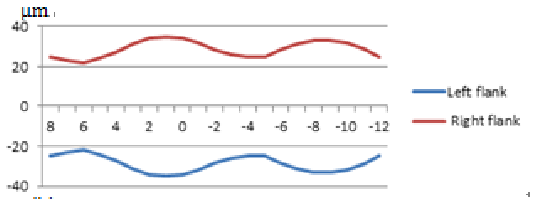
Figure 7 (b) shows well known profile errors in case of same run-out (ε). The theoretical value is 8.5 μm when the run-outs are 25 μm . The simulated value is a little bigger than theoretical one because of the cut off action as shown in figure 7 (a).

Figure 7 (c) is the profile errors correspond to study by Fieseler and Lausberg. [3] Both of results are mostly same.

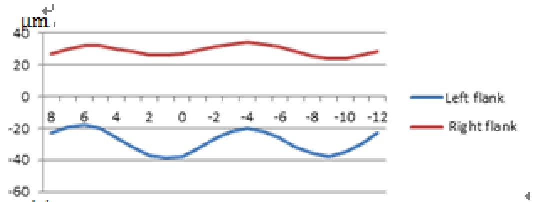
On a specified condition such that the run-out value of one side is a half of another side, a tooth profile is almost true involute as shown in figure 7 (d). The result is obtained when both of maximum run-out positions is on same plane including hob arbor axis. If the positions are not on the plane, both of profile errors are waved as shown in figure 7 (e).



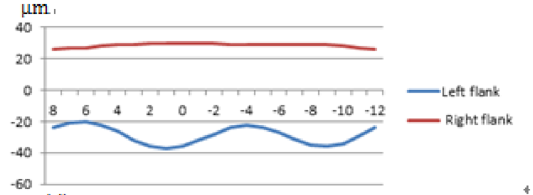
(a) $\varepsilon_1 = \varepsilon_2 = 0$ $\phi_{off} = 0$



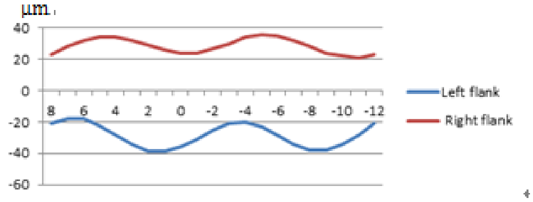
(b) $\varepsilon_1 = 0.025$ mm $\varepsilon_2 = 0.025\ mm$ $\phi_{off} = 0$



(c) $\varepsilon_1 = 0.025$ mm $\varepsilon_2 = 0\ mm$ $\phi_{off} = 0$



(d) $\varepsilon_1 = 0.025$ mm $\varepsilon_2 = 0.0125\ mm$ $\phi_{off} = 0$



(e) $\varepsilon_1 = 0.025$ mm $\varepsilon_2 = 0.0125\ mm$ $\phi_{off} = 90\ degree$

Figure 7: Simulated tooth profile error results related some run-outs conditions

Figure 8 shows the measured tooth profiles of a gear hobbed under the condition that the radial run-out $\varepsilon_1 =$ was 25 μm and ε_2 was 6 μm . The chart shows the tooth profiles are similar to the simulation in figure 7 (e).

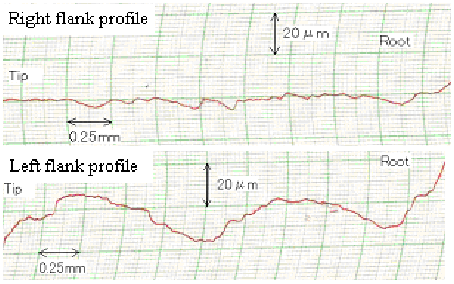


Figure 8: Inclination of hob axis in case of different run-out value $\varepsilon_1 = 25\,\mu m,$ $\varepsilon_2 = 6\mu m$

6 Conclusions

The relatively simple tooth profile error estimation method by checking the movement of only involute generating point on each cutting edge of the hob is applicable to know tooth profile errors caused by radial run-out of the hob. When a gear is cut under a condition that the radial run-out of one side is about twice of the other side, the right or left tooth profile is closed to involute curve. The tooth side depends on the side of the bigger run-out. However, if the positions which produce the maximum run-out are not on a same plane including hob arbor axis, the both side tooth profiles turns into the form which waves.

References

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