Glance on the Convergence of Godard Blind Equalization: Review

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Abstract
This paper studies the convergence of Godard blind equalization which based on least mean square (LMS) algorithm. It focuses on studying the effect of changing the step-size of LMS algorithm on the convergence of Godard algorithm. Simulation results show that the increase in step-size has negative impact on the convergence.

Keywords: *Godard blind equalization, LMS algorithm*

1 Introduction

Intersymbol interference (ISI) is a limiting factor in many communication environments. ISI can arise from bandlimited channels (telephone channels), and time-varying multipath fading channels. To achieve high-speed reliable communications, channel identification and equalization are necessary to overcome the effects of ISI. Traditionally, channel identification and equalization are achieved either by sending training sequences, or by designing the equalizer based on a priori knowledge of the channel. The latter approach is often not suitable for a radio communication environment since little knowledge about such a channel can be assumed a priori. The standard adaptive approach, though attractive in handling time-variant channels, has to waste a fraction of the transmission time for a training sequence.

In contrast to standard adaptive equalization methods, the so-called blind equalization methods do not require a training sequence. Instead, the statistical properties of the transmitted signals are exploited to carry out the equalization at the receiver without access to the symbols being transmitted. Blind equalization algorithms are essentially adaptive filtering algorithms designed in such a way that they do not need the external supply of a desired response to generate the error signal in the output of the adaptive equalization filter. In other words, the algorithm is blind to the desired response. However, the algorithm itself generates an estimate of the desired response by applying a nonlinear transformation on sequence involved in the adaptation process. There are three important families of blind equalization algorithms depending on where the nonlinear transformation is being applied on the data. These are:

(i) The Bussgang algorithms, where the nonlinearity is in the output of the adaptive equalization filter [1-4].
(ii) The polyspectra algorithms, where the nonlinearity is in the input of the adaptive equalization filter [5-6].
(iii) The algorithms where the nonlinearity is inside the equalization filter, i.e. nonlinear filter (e.g. Volterra) and neural network [7].

This paper focuses on one of the Bussgang

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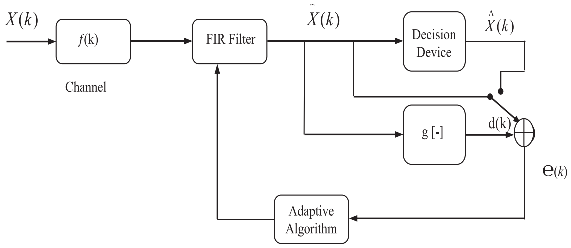

Figure 1: Bussgang block diagram.

algorithms known as Godard algorithm [2], also, called Constant Modulus Algorithm (CMA). Many researchers have contributed to Godard algorithm [8-13].

Figure 1 shows the general block diagram of Bussgang techniques. The important part in this diagram is the nonlinear function [-] appeared in the output of the equalization filter. Also, two important parts shown in Figure 1 are adaptive algorithm and FIR filter which are considered in the next sections.

2 Adaptive Algorithm

Adaptive algorithms are self-adjusting or self-designing techniques that can be applied to the analysis of signals with known, or time-varying, statistics. The adaptive algorithm starts with a set of initial conditions which, after successive iterations, converges to an optimum solution, provided that the signal is stationary. On the other hand, if the signal has time-varying statistics (i.e., it is nonstationary), the adaptive algorithm exhibits tracking capabilities by following up the variations in the statistics of the signal.

The adaptive algorithms for the operation of adaptive filters can be identified in three distinct categories:

(i) Approaches based on WIENER filter theory.
(ii) Approaches based on KALMAN filter theory.
(iii) The method of least squares.

These algorithms are based on mean square error (MSE) or least squares criteria, and consequently, employ second order statistics of the signal involved in the adaptation process and are limited to tracking variations in the second order statistics domain.

The third category (method of least squares) is a widely used technique for optimizing adaptive filters. Its popularity stems from its simplicity and power, and from the present widespread application of digital signal processing techniques such as finite impulse response (FIR) filtering. Thus, one of these algorithms was chosen to be studied, which is ***Least Mean Square Algorithm (LMS)***.

There is a fourth category of adaptive filtering algorithms that has received a lot of attention recently. These algorithms are well suited for problems where tracking of higher-order statistical variations is needed. This class of techniques is based on higher-order statistics (HOS) or nonlinear (e.g. non-MSE) criteria.

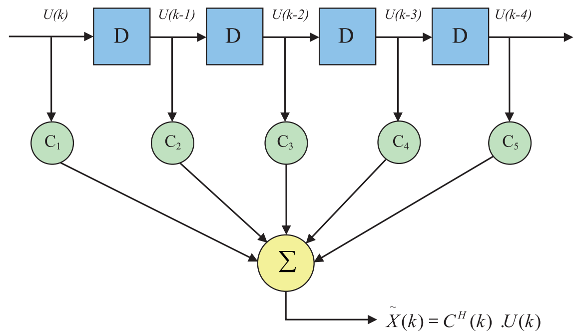

Figure 2: FIR Filter architecture.

3 Architecture of FIR Filter with LMS Adaptation

A common architecture of FIR filter is the tapped delay line shown in Figure 2. The Ds represent delay elements, or registers, that also store the input sample values *U(k) = [U(k), U(k-1), U(k-2),…..]*. Each time a new sample is input, the other samples in the filter are shifted to the right until they encounter no further delay elements. The FIR filter in this figure can hold five samples at any one time. This is called a five-tap FIR filter. Typical FIR filter lengths range from 3 to over 100 taps, depending on the application. Each sample that is in the filter is multiplied by a tap weight, or "coefficient," $C(k) = [c_1, c_2, c_3, ….]$. The products of all of the multiplications are summed to provide the filter's output, $\tilde{X}(k) = C^H(k)\ .U(k)$, where H denotes the transpose conjugate.Regardless of the implementation of the FIR filter, all of the filters have the same problem of determining the tap weights.

Determining the optimal tap weights is performed by the LMS algorithm. Specifically, LMS adjusts the tap weights of the filter to minimize the mean square error at the output of the FIR filter. Figure 3 shows how simply the LMS update equation fits into the flow of the tapped delay line FIR filter. Specifically, the output

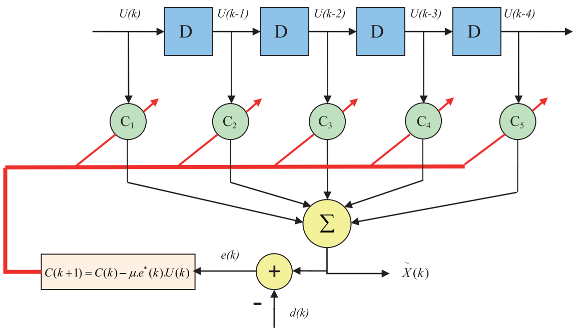

Figure 3: Architecture of FIR Filter with LMS adaptation.

of the filter is used to determine the error term $e(k)$ as follows

$$e(k) = \tilde{X}(k) - d(k) \tag{1}$$

Where $\tilde{X}(k) = C^{H}(k)\,.U(k)$ is the equalizer output and $d(k)$ is the desired ISI-free response.

The LMS update equation then uses the error term and the previous tap weights to generate the new tap weights as

$$C(k+1) = C(k) - \mu.e^{*}(k).U(k) \tag{2}$$

Where $e^{*}(k)$ is the complex conjugate of $e(k)$, μ is LMS update parameter, also, call step-size, and $U(k)$ is the input of the filter.

Three important factors that govern the adaptation process of the LMS algorithm are:

- Initial tap weights setting.
- Error term determination.
- LMS step-size, μ

Setting the initial tap weights is critical. If the taps are set close to their optimal values, the adaptation will quickly converge to the "best" tap weights. Further, the output of the FIR filter will be clean and accurate, beginning with the first sample. If the taps are set far from their optimum values, it will take longer to adapt the coefficients to useful values that can provide good signals at the output of the FIR filter. In order to drive the LMS update equation, it is necessary to determine the error between the output of the FIR filter and a desired target value (equation1). Further, the selection of the proper value for the LMS update parameter, μ, has a key role in determining the speed, accuracy, and stability of the filter adaptation.In general, a small value of step-size results in slower adaptation.

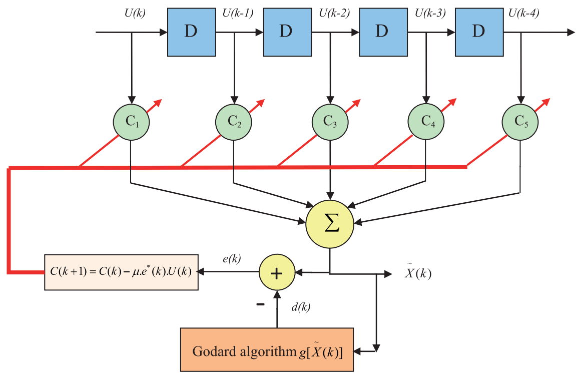

Figure 4: Godard blind equalizer.

4 Architecture of Godard Blind Equalizer

In order to determine the desired values ($d(k)$), there exist a number of different algorithms. Godard algorithms are the most commonly employed algorithms. One of their more remarkable features is their simplicity. However their main drawback is that they usually require a high number of data symbols to achieve convergence.

General equation of Godard algorithm is as follows:

$$g[\tilde{X}(k)] = \frac{\tilde{X}(k)}{\left|\tilde{X}(k)\right|}\left\{\left|\tilde{X}(k)\right| + R_p\left|\tilde{X}(k)\right|^{p-1} - \left|\tilde{X}(k)\right|^{2p-1}\right\} \tag{3}$$

Where $R_p = \dfrac{E\left\{\left|X(k)\right|^{2p}\right\}}{E\left\{\left|X(k)\right|^{p}\right\}}$, X($k$) is the transmitted sample

Figure 4 shows the complete architecture of Godard blind equalizer.

4.1 *Constant Modulus Algorithm (CMA)*

Among Godard Algorithms family, Constant Modulus Algorithm (CMA) is the most used adaptive algorithm for blind channel equalization. CMA is the specific Godard algorithm for P=2 in equation 3.

CMA uses the constant modularity of the signal as the desired property. It assumes that the input to the channel is a modulated signal that has constant amplitude at every instant in time. Any deviation of the received signal amplitude from the constant value is considered a distortion introduced by the channel.

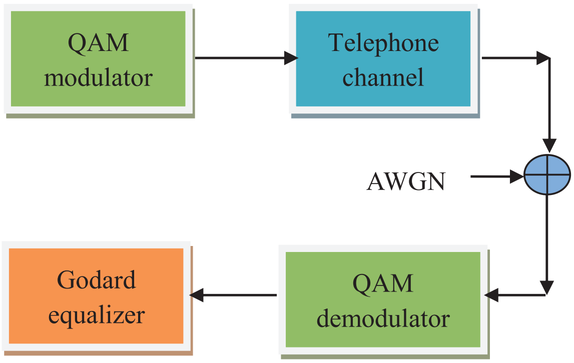

Figure 5: Communication system.

The distortion is mainly caused by band limiting or multi-path effects in the channel. Both these effects result in intersymbol-interference (ISI) as mentioned before, and thus distort the received signal. CMA attempts to remove these effects of the channel from the received signal by forcing the output of the adaptive filter (Equalizer) to be of constant amplitude. CMA can also be used for QAM signals where the amplitude of the modulated signal is not the same at every instant. The errore(k) is then determined by considering the nearest valid amplitude level of the modulated signal as the desired value.

5 Simulation Results

A series of computer simulation tests have been carried out on the communication system in Figure 5, which consists of QAM modulator operates at 9.6kb/s (with 16-ary rectangular constellation), telephone channel, Additive White Gaussian Noise (AWGN), QAM demodulator, and Godard equalizer. The signal-to-noise-ratio was considered to be 40dB in order to assure faster convergence. Three values of step-size were used, small value (0.000007), medium value (0.00007), and large value (0.0002).

Figure 6 shows the slow convergence of the mean square error for small step-size, while, Figure 7 shows the eye-diagram generated by the equalizer.

Figure 8 shows that the convergence becomes faster for medium value of step-size, but, at the expense of increasingthe instability, while, Figure 9 shows the eye-diagram which becomes less clear compared to the one for small step-size.

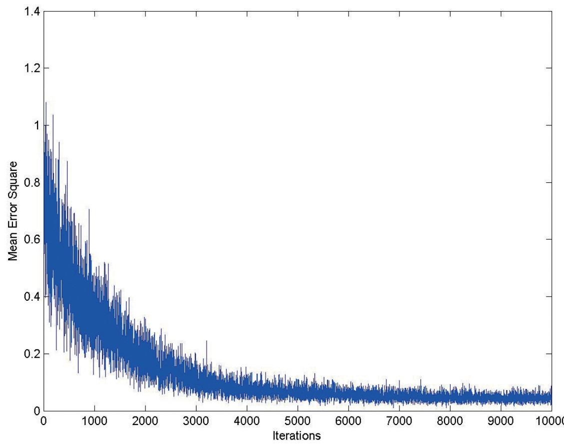

Figure 6: Mean square error for small step-size.

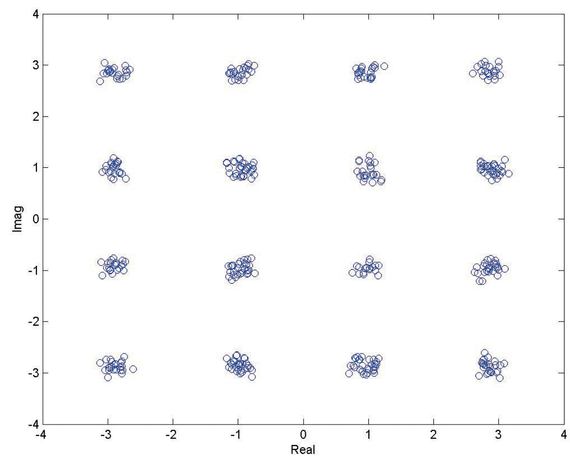

Figure 7: Eye-diagram for small step-size.

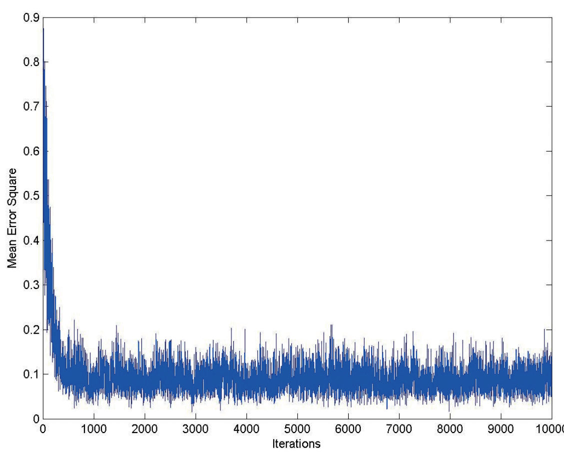

Figure 8: Mean square error for medium step-size.

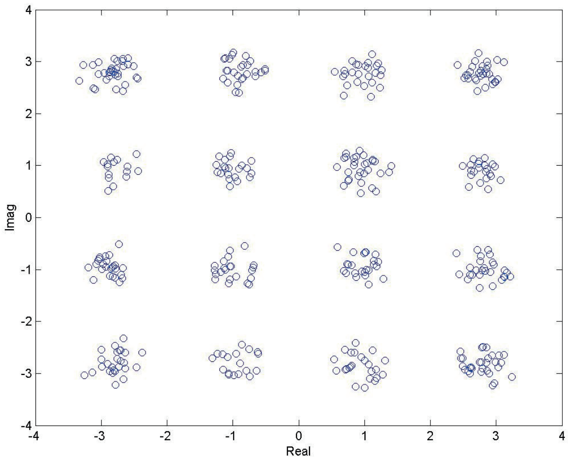

Figure 9: Eye-diagram for medium step-size.

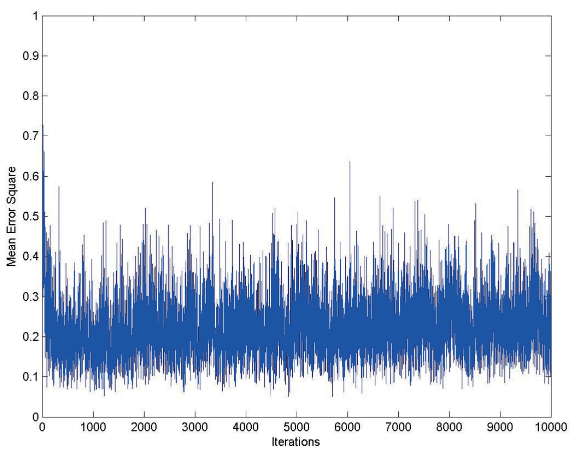

Figure 10: Mean square error for large step-size.

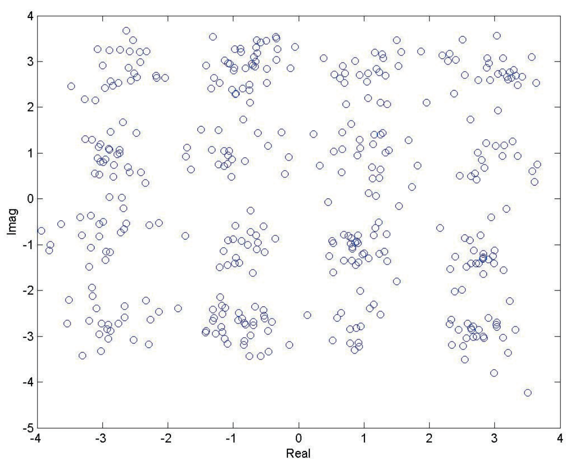

Figure 11: Eye-diagram for large step-size.

Figure 10 shows the extreme increase in the speed of convergence with extreme instability for large value of step-size, while, Figure 11 shows that the eye-diagram becomes unclear.

6 Conclusions

The blind equalization based on LMS criteria using constant modulus algorithm (CMA) has been tested using data transmission system at 9.6kb/s over telephone channel. The step size of the LMS algorithm, μ, was varied to see its effect on the convergence of the algorithm. From the results achieved it is apparent that as the step size, μ, increases, the system exhibits very fast adaptation but unstable response, while, when the step size becomes small, the adaptation goes to be slow but stable.

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